Competing for Talent

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Abstract

In many labor markets, e.g., for lawyers, consultants, MBA students, and professional sport players, workers get offered and sign long-term contracts even though waiting could reveal significant information about their capabilities. This phenomenon is called unraveling. We examine the link between wage bargaining and unraveling. Two firms, an incumbent and an entrant, compete to hire a worker of unknown talent. Informational frictions prevent the incumbent from always observing the entrant's arrival, inducing unraveling in *all* equilibria. We analyze the extent of unraveling, surplus shares, the average talent of employed workers, and the distribution of wages within and across firms.

Keywords: Unraveling, Talent, Wage Bargaining, Competition, Uncertainty.
JEL Codes: C7, D8, J3

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1 Introduction

On May 23$^{\text{rd}}$ 2015 Martin Ødegaard became the youngest player to represent Real Madrid. Nineteen months later, he was loaned to SC Heerenveen, a dutch football club ranked 441$^{\text{st}}$ worldwide. In many labor markets, talent is a scarce resource and an individual's true capability takes some time to be determined. Yet the demand for talent is steadily increasing.[1] Problems related to the timing of recruitment are therefore commonplace far beyond the football industry. It is for instance well documented that law firms and consultancies tend to recruit students long before graduation.[2] Presumably, efficiency could be improved if firms instead waited to know more about talent.

There exists a rich literature studying "unraveling", yet the interplay with wage bargaining remains largely unexplored. This is somewhat surprising since, if firms are uncertain about talent, then bargaining ought to be a key determinant of the timing of recruitment. The principal contribution of our paper is to shed light on the link between wage bargaining and unraveling. How does between-firm competition for talented workers determine unraveling? How is surplus shared among firms and workers? What is the relationship between the way surplus is shared and unraveling? What implications does this have for the distribution of wages within and across firms?

Our stylized model has the following features. Two firms A (the incumbent) and B (the entrant) compete to hire a worker of unknown talent. Competition is measured by the intensity of firm B's (random) arrival process. Informational frictions prevent A from always observing B's arrival. Both firms make take-it-or-leave-it offers to the worker, that can be revised over time. As time goes by, the market gradually learns the talent of the worker. The game ends the instant the worker accepts an offer from one of the two firms.

The role of informational frictions in wage bargaining has been emphasized on numerous occasions in the search literature. Two opposing views have been expressed. Burdett and Coles (2003) argue that outside offers are not verifiable and are therefore ignored by the current firm.[3] By contrast, Cahuc, Postel-Vinay and Robin (2006) provide empirical evidence that

[1] See Juhn, Murphy and Pierce (1993), Autor, Levy and Murnane (2003), Bresnahan, Brynjolfsson and Hitt (2002), or Helpman, Itskhoki, Muendler and Redding (2017).

[2] See Roth (1984), Roth and Xing (1994), Avery, Jolls, Posner and Roth (2001), or Fréchette, Roth and Ünver (2007).

[3] In the setting we analyze, the role of this feature is to impart bargaining power to the entrant. In a similar spirit Mortensen (2005) points out that "Making counter-offers is not the norm in many labour markets. More typically, a worker who informs his employer of a more lucrative outside option is first congratulated and then asked to clear out immediately". Similarly, Shimer (2006) considers a bargaining protocol in which as soon

counter-offers do matter for wage determination. The framework we propose aims to reconcile both views by adding informational frictions to a simple search model: when informational frictions are small firms effectively Bertrand compete for the worker, and when these frictions are large, the worker automatically gets hired by the entrant. We find that informational frictions have very important implications regarding equilibrium unraveling. Among other things, absent informational frictions unraveling never occurs.

The model's efficient outcome balances the gain from waiting to learn the worker's realized talent versus the loss from delaying hiring in case the worker is in fact talented. By contrast, the basic tension underlying the non-cooperative outcome is the following. On the one hand, the expectation of future competition from B creates an incentive for A to make generous offers to the worker early on. On the other hand, the worker's reservation wage internalizes future competition. By pushing the reservation wage above the worker's current expected productivity, the latter effect induces the incumbent to wait before making offers acceptable to the worker.

We show that all equilibria exhibit some amount of unraveling: with some probability the worker is hired before the social optimum. The intuition is that if B's arrival occurs before the worker has accepted an offer from A then part of the social surplus transfers over to B. Consequently, relative to the social planner's point of view, from the worker and firm A's perspective the gain from waiting to learn the worker's realized talent is strictly lower. The worker and firm A thus reach an agreement too early. This mechanism emphasizes the role of informational frictions. Absent informational frictions, if upon B's arrival the worker were still unemployed then the entire social surplus would go to the worker, in which case the worker and firm A's incentives would be aligned with those of the social planner.

Our results also highlight a novel trade-off between unraveling and inequality: the more equal the division of surplus between the incumbent and the worker, the greater the extent of unraveling. Intuitively, when bargaining power is distributed evenly between the worker and firm A then B is able to claim a large part of the surplus. This in turn lowers the worker and firm A's gain from waiting to learn the worker's realized talent. We find in addition that the trade-off between unraveling and inequality becomes more severe when informational frictions increase.

While the model exhibits multiple equilibria (which differ regarding the players' respective shares of the surplus), interestingly the time at which firm A makes its first acceptable offer

as a worker meets a new firm, the worker first decides whether to switch or not and then bilaterally bargains with the new firm.

to the worker is the same in all equilibria. Characterizing this time enables us to quantify the extent of unraveling, and to make sharp predictions regarding the effect of various parameters. We show for instance that the basic tension at the heart of our model induces a non-monotonic relationship between competition and unraveling. When competition is low, increasing competition worsens unraveling. When competition is high, more competition reduces unraveling. Hence, unraveling is maximized when competition is intermediate.

We analyze the extent of unraveling at firm B relative to firm A. Counter to intuition, unraveling can be worse at B than at A. The reason is that anytime B's arrival is unobserved by A, then the worker's upside potential from waiting instantly drops, as does the worker's reservation wage. When offers from A are greater than the new reservation wage, B is then forced to hire the worker without further delay. Interestingly, in equilibrium, workers of identical talent can be hired at varying wages both within a given firm, and across the two firms. This finding is consistent with empirical evidence on residual wage dispersion that finds that workers with the same education receive different wages.[4] Finally, we show that while more competition benefits the worker and hurts A, firm B can be better or worse off. By contrast, informational frictions always benefit the firms and hurt the worker. Our results can also be applied to tenure contracts that are common in law and consulting firms, as well as in academia.

The paper is organized as follows. Section 2 presents a simplified two-period example illustrating the basic tension at the heart of our analysis. The model is presented in Section 3. Sections 4 and 5 contain some preliminaries. The core analysis is presented in Section 6. The comparative statics results are in Section 7. Section 8 concludes.

Related Literature. We contribute to the literature on unraveling by exploring the link between wage bargaining and unraveling. Unlike this paper, the literature has focused for the most part on matching markets (see recent contributions by Echenique and Pereyra (2016) and Du and Livne (2016) for an overview of this strand of research). In Li and Rosen (1998) and Li and Suen (2000) contracting early on acts as an insurance device by which individuals reduce the risks implied by matching after all uncertainty has been resolved. In our setting, insurance plays no role since all parties are risk-neutral. Our model is closer to Damiano, Li and Suen (2005) and Ambuehl and Groves (2017). Damiano et al. (2005) develop a friction-based theory of unraveling. However, the mechanism they identify is very different from ours since there is

[4]See Lemieux (2006), or Firpo, Fortin and Lemieux (2009).

no uncertainty in their framework.[5] Like us, Ambuehl and Groves (2017) examine a setting in which talent is learned along the way, and where firms make early offers in an attempt to avoid future competition. However, the focus of their paper is different from ours, and there is no wage bargaining in their setting.[6]

Our work is closely linked to the non-cooperative bargaining literature with symmetric information about an uncertain surplus and more than two players pioneered by Merlo and Wilson (1995). The authors show that efficiency obtains as long as all players are required to agree on a sharing rule. Allowing for general voting rules, Eraslan and Merlo (2002) show that (a) unraveling can occur and (b) multiple equilibria can exist. Rather than institutional voting rules, in our framework informational frictions are the drivers of unraveling.

Three recent papers study the impact of learning and/or potential entry on bargaining with one-sided asymmetric information. Fuchs and Skrzypacz (2010) study a model in which outside options arrive at a stochastic date, while Daley and Green (2017) study bargaining where the parties obtain information about the uncertain surplus through an exogenous news process. Lomys (2017) analyzes the interaction of learning and potential entry in bargaining. The focus of these papers is on Coasean forces, and trade is inefficiently delayed. In sharp contrast trade occurs inefficiently early in our setting.

Finally, as noted earlier, our paper is closely related to the on-the-job search literature, in which the role of outside offers in wage bargaining occupies a prominent place.[7] More broadly, our work contributes to the employer learning literature (e.g., Farber and Gibbons (1996), Altonji and Pierret (2001), Lange (2007), Kahn and Lange (2014)). Unlike those papers, our focus is on learning prior to hiring, and we abstract away from the issue of investment into talent explored in Acemoglu and Pischke (1998) and Autor (2001) among others.

[5] The market in Damiano et al. (2005) operates in two rounds, and agents incur a cost for each round in which they participate. The unique equilibrium is such that no one participates in the second round, and all agents accept to match in the first round.

[6] In a similar spirit, Halaburda (2010) obtains unraveling as less attractive firms offer early contracts, which workers accept due to the fact that they are on the long side of the market and may otherwise never get a job.

[7] In addition to the papers cited earlier, our paper is somewhat related to Moscarini et al. (2004). There, the degree to which a firm commits not to compete is called "corporate culture". The fact that it may be profitable for firms to commit ex ante not to match outside offers has also been pointed out by Postel-Vinay and Robin (2004). In that spirit, one possible interpretation is to view the informational frictions in our framework as a commitment device for the incumbent firm.

2 A Simple Two-Period Example

The following two-period example highlights the main trade-off of the model. Consider two firms, A and B, and one worker interacting over two periods. The worker is unemployed at the beginning of period 1, with unknown talent, high (H) or low (L). Firm A (the incumbent) arrives at $t = 1$. With probability $\lambda \in (0, 1)$ firm B (the entrant) arrives at $t = 2$, and with probability $1 - \lambda$ firm B never arrives. For instance, the worker might be a trainee at firm A, and firm B a competing firm looking to fill an opening position. Ex-ante the probability of H is p_0. A H-worker produces $b > 0$ and a L-worker produces $-c < 0$. A firm produces zero if it does not hire the worker. The worker's talent is publicly revealed at $t = 2$, but if firm A waits and hires the worker at $t = 2$ then both firm A and the worker incur a cost $r > 0$. We assume that waiting is socially optimal, i.e., $(1 - p_0)c > 2r$.[8] The worker has no intrinsic preference for any of the two firms, which compete on the wage in order to hire the worker. Each period until the worker is hired, firm A makes a take-it-or-leave-it (TIOLI) offer to the worker. If the first offer is rejected the game moves on to period 2. We assume that with probability $\beta \in (0, 1)$ informational frictions prevent A from verifying B's arrival. The bargaining protocol in period 2 is as follows. If A observed B's arrival, then A and B simultaneously make TIOLI offers to the worker. If B's arrival was unobserved, then A makes its offer first; B observes the offer from A, and follows with a counter-offer.

We uncover the unique equilibrium by backward induction. At $t = 2$, either A observes B or it does not. If talent is H and A observes B, then both firms simultaneously offer b. However, if A does not observe B, then each firm offers 0.[9] Hence, the worker's expected payoff from rejecting an offer at $t = 1$ is given by

$$\hat{w} = \lambda(1 - \beta)p_0 b - r;$$

$\hat{w}$ is also the worker's reservation wage at $t = 1$.

We now examine $t = 1$. Firm A can either offer $\hat{w}$, or make an offer which the worker rejects. In the first case, the expected payoff of A is $[p_0 b - (1 - p_0)c] - [\lambda(1 - \beta)p_0 b - r] = r - (1 - p_0)c + [(1 - \lambda)(1 - \beta) + \beta]p_0 b$; in the second case, the expected payoff of A is $(1 - \lambda)p_0 b - r$. It ensues that A makes an offer which the worker rejects if and only if

[8] The social gain from waiting until $t = 2$ is c if the worker's talent is L, whereas the social cost is $2r$.

[9] If A does not observe B, then no (strictly) positive offer is optimal from A's perspective: either B has not arrived and then 0 is acceptable for the worker, or B has arrived in which case A must offer more than b in order to hire the worker.

$r - (1-p_0)c + [(1-\lambda)(1-\beta) + \beta]p_0 b < (1-\lambda)p_0 b - r$. In other words, efficiency obtains if and only if:

$$\underbrace{\big[(1-p_0)c - 2r\big]}_{\text{efficiency gain}} - \underbrace{\beta\lambda p_0 b}_{\text{expected payoff of B}} > 0.$$

The condition above can be decomposed as follows. The term inside the first square bracket represents the efficiency gain from waiting until $t = 2$. The term inside the second square bracket represents the share of surplus accruing to firm B at $t = 2$ but evaluated at $t = 1$. The difference between the first and second square brackets thus represents the worker and firm A's joint incentive to wait until $t = 2$.

The simple two-period example examined above illustrates well the basic tension at the heart of our model. On the one hand, future competition from B creates an incentive for A to hire the worker early on: the expected payoff of A from waiting until $t = 2$ is $(1-\lambda)p_0 b - r$, which is decreasing in λ. On the other hand, the worker's reservation wage internalizes future competition: $\hat{w}$ is increasing in λ. Whenever $\hat{w}$ is greater than the worker's current expected productivity, $p_0 b - (1-p_0)c$, the latter effect induces A to wait until $t = 2$ before making an offer acceptable to the worker. Finally note that, absent informational frictions (that is, for $\beta = 0$), efficient always obtains.

This two-period example is unfortunately too stylized to capture some crucial aspects of the more general setting. Whereas with two periods the worker's reservation wage is equal to 0 in the last period, with an infinite (time) horizon the worker's reservation wage is always strictly greater than 0 as long as the worker's talent remains uncertain. As we will see, this feature turns out to have profound consequences. First, the infinite-horizon model exhibits multiple equilibria, which differ regarding efficiency and the players' respective shares of the surplus. Second, whereas in the two-period example increasing competition always worsens unraveling, in general the relationship between competition and unraveling is in fact non-monotonic. Finally, we show that for unraveling to occur, informational frictions are both necessary *and* sufficient in the infinite-horizon model.

3 Model

The model is an infinite-horizon analogue of the two-period example studied in Section 2. Time is discrete and denoted by $t \in \mathbb{T}_\Delta := \{0, \Delta, 2\Delta, \dots\}$ where $\Delta > 0$ is the length of a period. We next lay out the details of the model.

Talent and Payoffs. The worker's talent is $\omega \in \{H, L\}$: the worker produces $b > 0$ if $\omega = H$ and $-c < 0$ if $\omega = L$. The probability that the worker's talent is H is given by $p_0 \in (0, 1)$. The worker's payoff from unemployment is normalized to 0; her (undiscounted) payoff from becoming employed equals her wage, w.[10] A firm's payoff from hiring the worker is $b - w$ if $\omega = H$ and $-c - w$ if $\omega = L$. A firm's payoff is normalized to 0 if it does not hire the worker. In every period, the discount factor of all players is given by $e^{-r\Delta}$.

Learning. Learning about talent occurs through the worker's slip-ups: if her talent is H the worker never slips up whereas if $\omega = L$ each period the probability of a slip-up is given by $1 - e^{-\eta\Delta}$, where $\eta > 0$. Let p_t denote the posterior probability assigned to $\omega = H$ after observing the number of slip-ups having occurred by t. Applying Bayes' rule, $p_t = 0$ after the first slip-up; on the other hand, as long as the worker does not slip up, in the limit as $\Delta \to 0$, p_t evolves according to the differential equation

$$\dot{p}_t = \eta p_t(1 - p_t). \tag{1}$$

Thus, in particular, p_t is increasing in t conditional on no slip-up. To avoid repetitions, all derivatives with respect to time will be understood conditional on no slip-up. The expected (undiscounted) social surplus from hiring the worker at time t is denoted S_t, that is,

$$S_t := S(p_t) := p_t b - (1 - p_t)c.$$

It is easy to check that in the limit (as $\Delta \to 0$) S_t evolves according to $\dot{S}_t = \eta(1 - p_t)(S_t + c)$.

As will become clear later (see Remark 1 at the end of this section), in the types of environment we examine, private information of the incumbent plays limited role. For expository simplicity we therefore assume that learning is public, that is, the worker's slip-ups are observed by all players. Thus, throughout the game, p_t represents the players' common belief that $\omega = H$, henceforth simply referred to as the *belief.*

Competition and Informational Frictions. The arrival time of the entrant, denoted $T_B \in \mathbb{T}_\Delta$, is a random variable. If firm B has not arrived by t, the probability that firm B will arrive next period is given by $1 - e^{-\lambda\Delta}$, where $\lambda \in (0, \infty)$. Hence T_B/Δ is geometrically distributed with success rate $1 - e^{-\lambda\Delta}$ (and, in the limit, T_B is exponentially distributed with

[10] Since hiring is irreversible, we can think of w as the net present value of the worker's future income stream.

parameter λ). Informational frictions can prevent A from observing B's arrival. Specifically, B's arrival is concealed from A with probability $\beta \in (0, 1)$ and observed by A with probability $1 - \beta$.[11] For all t, define the random variable s_t taking a value in $\{s_0, s', s''\}$, such that $s_t = s_0$ if B has not yet arrived by t (that is, if $T_B > t$), $s_t = s'$ if B's arrival was unobserved by A, and $s_t = s''$ if B's arrival was observed by A. Note that by construction firm A only observes whether $s_t \in \{s_0, s'\}$ or $s_t = s''$. We assume on the contrary that the worker and firm B observe s_t. With a slight abuse of terminology, we will refer to s_t as the *state*.

Timing. The timing within period $[t, t + \Delta)$ is as follows:

- *if* $s_t = s_0$: firm A makes an offer; the worker then chooses whether to accept or reject the offer;

- *if* $s_t = s'$: firm A makes an offer which is observed by B; B then makes a counter-offer and the worker chooses whether to accept one offer or to reject both;

- *if* $s_t = s''$: A and B simultaneously make an offer; the worker then chooses whether to accept one offer or to reject both.

The game ends the instant the worker accepts an offer from one of the two firms (at which point payoffs are realized).

Strategies and Equilibrium. We focus on pure stationary strategies. Specifically, a strategy for A specifies a mapping $w_A(p_t, \{s_0, s'\})$ representing the wage offer as a function of p_t when the state belongs to $\{s_0, s'\}$ and a mapping $w_A(p_t, s'')$ representing the wage offer in state s''.[12] Similarly, a strategy for B specifies a mapping $w_B(p_t, w_A, s')$ representing the counter-offer in state s' as a function of p_t and firm A's current offer w_A, and a mapping $w_B(p_t, s'')$ representing the wage offer in state s''. A strategy for the worker is a mapping $d : [p_0, 1] \times \{s_0, s', s''\} \times \mathbb{R}^2 \to \{A, B, \varnothing\}$, indicating which offer to accept (if any) as a function of p_t, the state, and current wage offers. To simplify the exposition we assume that both firms offer $-c$ whenever $p_t = 0$, which the worker always rejects. By (1), this enables us to focus on strategies defined over $p_t \in [p_0, 1]$ only.

Our equilibrium concept is sequential equilibrium in pure stationary strategies. Henceforth, we refer to such sequential equilibria as equilibria for short.

[11] The observability of B is drawn once and for all, for simplicity.

[12] As we make clear later, as long as $s_t \in \{s_0, s'\}$, firm A's beliefs concerning s_t are payoff-irrelevant for A. We therefore ignore them without loss of generality.

Remark 1. *All results in the paper continue to hold in the limit as $\Delta \to 0$ if, instead of public learning, firm A privately learned about ω by observing the worker's slip-ups while firm B learned about talent through the wage offers of the incumbent.*

4 Preliminary Analysis

We briefly analyze the social planner's problem, and use the planner's value function in order to construct a wage function playing a key role in the rest of the paper. The details of all derivations in this section are in Appendix B.

Let $V_\Delta(p)$, denote the planner's value function given initial belief p and period length Δ. If $\omega = L$ then not hiring the worker is the socially optimal decision. The planner's problem thus reduces to an optimal stopping problem, in which the stopping time determines the instant at which to hire the worker (at either one of the two firms) conditional on no slip-up having occurred. Let $t^*_\Delta \in \mathbb{T}_\Delta$ denote the planner's (unique) optimal stopping time given initial belief p_0.[13] Then:[14]

$$\begin{cases} \mathbb{E}_t\left[e^{-r\Delta}\max\{0, S(p_{t+\Delta})\}\right] - S(p_t) > 0 & \text{if } t < t^*_\Delta \\ \mathbb{E}_t\left[e^{-r\Delta}\max\{0, S(p_{t+\Delta})\}\right] - S(p_t) < 0 & \text{if } t \geqslant t^*_\Delta. \end{cases}$$

Intuitively, the marginal benefit to waiting an additional period of length Δ is positive if and only if $t < t^*_\Delta$. In order to make the bargaining problem non-trivial, we will assume throughout that $t^*_\Delta > 0$. To shorten notation, henceforth let p^*_Δ denote the belief at time t^*_Δ conditional on no slip-up having occurred before that time.

Whenever $p_t > 0$, the above observations allow us to write

$$V_\Delta(p_t) = \begin{cases} e^{-r(t^*_\Delta - t)}\left(p_t + (1-p_t)e^{-\eta(t^*_\Delta - t)}\right) S(p^*_\Delta) & \text{if } t < t^*_\Delta \\ S(p_t) & \text{if } t \geqslant t^*_\Delta. \end{cases}$$

We next use the planner's value function in order to construct an auxiliary wage function $\hat{w}_\Delta(p)$. Henceforth, with a slight abuse of notation, we denote by $\mathbb{E}_p$ the expected value given

[13] The existence and uniqueness of the optimal stopping time is proven in the appendix.

[14] We ignore the case in which the equality holds at exactly t^*_Δ since such cases are knife-edge.

initial belief p. With this notation,

$$\hat{w}_\Delta(p) := \mathbb{E}_p[e^{-r\hat{T}} V_\Delta(p_{\hat{T}})], \tag{2}$$

where $\hat{T}/\Delta$ is geometrically distributed with success rate $(1-\beta)(1-e^{-\lambda\Delta})$. The lemma below lists the basic properties of $\hat{w}_\Delta(\cdot)$.

Lemma 1. *The following properties hold:*

1. $\hat{w}_\Delta(p) < V_\Delta(p)$ *for all* $p > 0$,
2. $\hat{w}_\Delta(p)$ *is an increasing function of* p,
3. $\hat{w}_\Delta(\cdot)$ *is increasing in* λ *and decreasing in* β,
4. *there exists* $t^\sharp_\Delta$ *such that, whenever* $p_t > 0$:[15]
   (a) $\hat{w}_\Delta(p_t) > S(p_t)$ *if* $t < t^\sharp_\Delta$,
   (b) $\hat{w}_\Delta(p_t) < S(p_t)$ *if* $t \geqslant t^\sharp_\Delta$.

Properties 1-3 are easily obtained from definition (2). Property 4 will enable us to simplify the exposition by assuming henceforth that $t^\sharp_\Delta = 0$. Extending all of our results to $0 < t^\sharp_\Delta$ is straightforward, but makes the statement of some results unnecessarily tedious. The functions $S(p)$, $V(p) := \lim_{\Delta\to 0} V_\Delta(p)$ and $\hat{w}(p) := \lim_{\Delta\to 0} \hat{w}_\Delta(p)$ are all illustrated in Figure 1. We also indicate the values $p^\sharp := \lim_{\Delta\to 0} p^\sharp_\Delta$ and $p^* := \lim_{\Delta\to 0} p^*_\Delta$. The next lemma provides an alternative representation of the function $\hat{w}_\Delta(p)$.

Lemma 2. *If* $\mathcal{T}/\Delta$ *is geometrically distributed with success rate* $(1 - e^{-\lambda\Delta})$ *then*

$$\hat{w}_\Delta(p) = \mathbb{E}_p[e^{-r\mathcal{T}}(1-\beta)V_\Delta(p_\mathcal{T}) + e^{-r\mathcal{T}}\beta\hat{w}_\Delta(p_\mathcal{T})]. \tag{3}$$

Equation (3) enables us to view $\hat{w}_\Delta(p)$ as the value of a lottery paying out $V_\Delta(\cdot)$ with probability $(1-\beta)$ and paying out $\hat{w}_\Delta(\cdot)$ with probability β, at a random time distributed like the random arrival time T_B of the entrant. This observation will be the cornerstone of the benchmark equilibrium construction we undertake in the next subsection.

[15] We ignore the knife-edge case in which $\hat{w}_\Delta(p_t) = S(p_t)$ at exactly $t = t^\sharp_\Delta$.

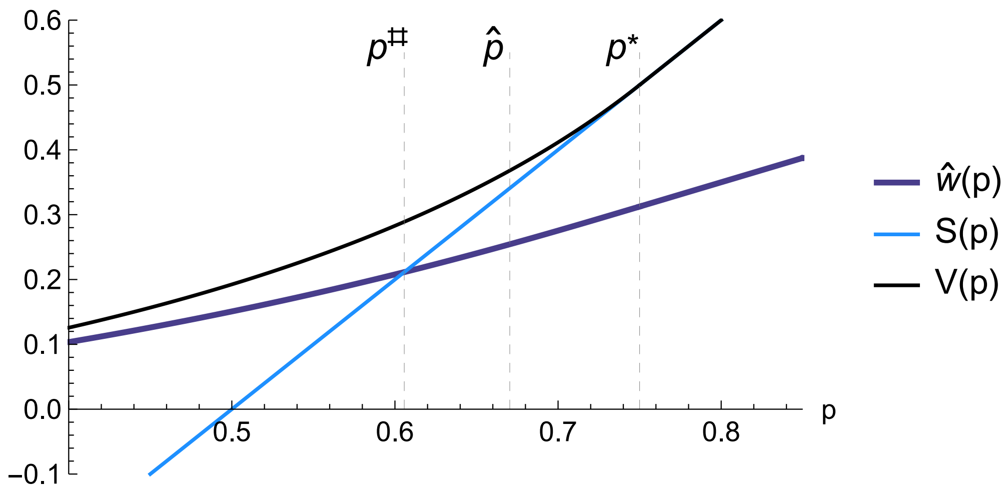

FIGURE 1

5 Two Simple Equilibria

In this section we examine two salient equilibria. The insights developed in this section will help us draw the contour of the core analysis undertaken in the subsequent section. The proofs for this section are in Appendix C.

5.1 Benchmark Equilibrium

The aim in this subsection is to construct an equilibrium in which:

- as long as $p_t > 0$ each firm offers $\hat{w}_\Delta(p_t)$ whenever the state belongs to $\{s_0, s'\}$, and the entire surplus $S(p_t)$ in state s'',
- in state s_0 the worker rejects all offers until a cutoff time $\hat{t}_\Delta \leqslant t^*_\Delta$,
- in state s' the worker accepts an offer immediately, and in state s'' the worker rejects all offers until t^*_Δ.

We denote this equilibrium by $\mathcal{E}_\Delta$ and call it the benchmark equilibrium. The cutoff time $\hat{t}_\Delta \in \mathbb{T}_\Delta$ is defined as the unique solution to[16]

$$\begin{cases} \mathbb{E}_t \left[e^{-(r+\lambda)\Delta} \max\{0, S(p_{t+\Delta}) - \hat{w}_\Delta(p_{t+\Delta})\} \right] > S(p_t) - \hat{w}_\Delta(p_t) & \text{if } t < \hat{t}_\Delta \\ \mathbb{E}_t \left[e^{-(r+\lambda)\Delta} \max\{0, S(p_{t+\Delta}) - \hat{w}_\Delta(p_{t+\Delta})\} \right] < S(p_t) - \hat{w}_\Delta(p_t) & \text{if } t \geqslant \hat{t}_\Delta. \end{cases}$$

Henceforth, let $\hat{p}_\Delta$ denote the belief at the time $\hat{t}_\Delta$ conditional on no slip-up having occurred before that time. Observe that $\hat{t}_\Delta$ is the optimal stopping time of a social planner with (virtual) surplus $S - \hat{w}_\Delta$ and discount rate $r + \lambda$, that is,

$$\hat{t}_\Delta = \arg\max_{T \in \mathbb{T}_\Delta} \mathbb{E} \left[e^{-(r+\lambda)T} \max\{0, S(p_T) - \hat{w}_\Delta(p_T)\} \right].$$

One shows that $\hat{t}_\Delta \leqslant t^*_\Delta$, with strict inequality when $\Delta > 0$ is sufficiently small. Again, to make the problem interesting, we will assume throughout that $\hat{t}_\Delta > 0$.[17]

For expository purposes in constructing the equilibrium strategy profile of this subsection, define

$$\underline{w}_\Delta(p_t, s') := \mathbb{E}_t \left[e^{-r\Delta} \hat{w}_\Delta(p_{t+\Delta}) \right].$$

This is the reservation wage given that, in all future periods, only wages corresponding to $\hat{w}(\cdot)$ are offered as long as $p_t > 0$. Note that, by the definition of $\hat{w}_\Delta(\cdot)$: $\underline{w}_\Delta(p_t, s') < \hat{w}_\Delta(p_t)$ for all $p_t > 0$. We can now state this subsection's result.

Proposition 1. *The following strategy profile, $\mathcal{E}_\Delta$, constitutes an equilibrium:*[18]

- $w_A(p_t, s'') = S(p_t)$ *and* $w_A(p_t, \{s_0, s'\}) = \hat{w}_\Delta(p_t)$.
- $w_B(p_t, s'') = S(p_t)$ *and*

$$w_B(p_t, w_A, s') = \begin{cases} \min\{w_A, S(p_t)\} & \text{if } t < t^*_\Delta \\ \max\left\{\underline{w}_\Delta(p_t, s'), \min\left\{w_A, S(p_t)\right\}\right\} & \text{if } t \geqslant t^*_\Delta. \end{cases}$$

[16] The existence of such a $\hat{t}_\Delta$ is an immediate consequence of Corollary 1 in the appendix. We rule out the non-generic case in which equality occurs at time $\hat{t}_\Delta$.

[17] When $\hat{t}_\Delta = 0$, trade will take place without delay. When $\hat{t}_\Delta > 0$, trade will inevitably exhibit delay in all equilibria.

[18] Recall that for expositional simplicity strategies are defined over $p_t \in [p_0, 1]$ only, that is, conditional on no slip-up having occurred.

•

$$d(p_t, s_0, w_A) = \begin{cases} A & \textit{if } t < \hat{t}_\Delta \textit{ and } w_A > \hat{w}_\Delta(p_t),\ \textit{ or } t \geqslant \hat{t}_\Delta \textit{ and } w_A \geqslant \hat{w}_\Delta(p_t) \\ \varnothing & \textit{otherwise} \end{cases}$$

$$d(p_t, s', w_A, w_B) = \begin{cases} A & \textit{if } w_A > w_B \geqslant \underline{w}_\Delta(p_t, s'),\ \textit{ or } w_A \geqslant \underline{w}_\Delta(p_t, s') > w_B \\ B & \textit{if } w_B \geqslant \max\{\underline{w}_\Delta(p_t, s'), w_A\} \\ \varnothing & \textit{otherwise} \end{cases}$$

$$d(p_t, s'', w_A, w_B) = \begin{cases} A & \textit{if } w_A \geqslant w_B \geqslant V_\Delta(p_t) \\ B & \textit{if } w_B > w_A \geqslant V_\Delta(p_t) \\ \varnothing & \textit{otherwise.}^{19} \end{cases}$$

Below, we sketch the main arguments showing that these strategies comprise an equilibrium. When the state is s'', the firms compete a la Bertrand. In this case, the worker commands the entire surplus and waiting until t^*_Δ is therefore the worker's optimal policy. By construction, the worker's reservation wage in the state s' is equal to $\underline{w}_\Delta(p_t, s')$. Consider next the state s_0, and let $\underline{w}_\Delta(p_t, s_0)$ denote the worker's reservation wage as a function of p_t in that state.[20] We can show that if the worker rejects an offer at time t, she can then do no better than to wait for B in the hope that the entrant's arrival will be observed by A, and to fall back on $\hat{w}_\Delta(p_{T_B})$ if this turns out not to be the case. As this policy yields expected discounted payoff $\hat{w}_\Delta(p_t)$ (see (3)), we obtain $\underline{w}_\Delta(p_t, s_0) = \hat{w}_\Delta(p_t)$. Thus, with the worker's strategy as in the statement of the proposition, the worker never accepts an offer below her reservation wage. The worker's strategy is thus a best response.

We next argue that each firm behaves optimally. Consider first firm B in state s'. As in the absence of a counter-offer any offer of $w_A \in \left[\underline{w}_\Delta(p_t, s'), S(p_t)\right]$ is accepted by the worker, in this case the best response of B is to match the offer from A. Any offer of $w_A < \underline{w}_\Delta(p_t, s')$ on the other hand is rejected by the worker. In this case, the best response of B is to instantly hire the worker at a wage of $\underline{w}_\Delta(p_t, s')$ if and only if $S(p_t) - \underline{w}_\Delta(p_t, s') \geqslant e^{-r\Delta}\mathbb{E}_t\left[\max\{0, S(p_{t+\Delta}) - \underline{w}_\Delta(p_{t+\Delta}, s')\}\right]$, which, for Δ sufficiently small, happens exactly when $t \geqslant t^*_\Delta$.

[19] In the defined equilibrium, when $w_A = w_B \geqslant V_\Delta(p_t)$ in state s'', it does not matter which offer the worker accepts.

[20] The worker's reservation wage is defined as the sup of the worker's expected discounted payoff assuming that the worker rejects the current offer(s).

Consider now firm A. In state s', any offer less than $S(p_t)$ is matched by B; this implies that the only way A can hire the worker is by offering more than the full surplus. Consequently, to show that firm A's policy is optimal when the state belongs to $\{s_0, s'\}$, it is enough to show that it is optimal conditional on the state s_0. Since $\underline{w}(p_t, s_0) = \hat{w}(p_t)$, firm A's problem in the state s_0 is akin to that of the social planner's with (virtual) surplus $S - \hat{w}$ and discount rate $r + \lambda$. By a previous remark, firm A's optimal policy is therefore to hire the worker at the cutoff time $\hat{t}_\Delta$.

5.2 Efficient Equilibrium

The benchmark equilibrium of the previous subsection is open to criticism since firm A offers $\hat{w}_\Delta(p_t)$ at all times $t < \hat{t}_\Delta$ although the firm strictly prefers to wait until $\hat{t}_\Delta$ is reached before hiring. Equilibrium is sustained only by the belief that the worker will in fact reject all offers from A before $\hat{t}_\Delta$ is reached. Moreover, the promise of receiving offers $\hat{w}_\Delta(p_t)$ at all future dates (conditional on no slip-up) raises the worker's current reservation wage, which in turn lowers the incumbent's maximum share of the surplus. One can argue that a more natural equilibrium would therefore entail A offering a wage equal to 0 until the time at which it actually wants to hire the worker. The next proposition characterizes such an equilibrium, which, as we show in Section 6, turns out to be the most efficient equilibrium. Before stating the result, define for the rest of this subsection:

$$\underline{w}_\Delta(p_t, s_0) := e^{-r(\hat{t}_\Delta - t)}\mathbb{E}_t\left[\left(1 - (1-\beta)\left(1 - e^{-\lambda(\hat{t}_\Delta - t)}\right)\right)\hat{w}_\Delta(p_{\hat{t}_\Delta}) + (1-\beta)(1 - e^{-\lambda(\hat{t}_\Delta - t)})V_\Delta(p_{\hat{t}_\Delta})\right]$$

if $t < \hat{t}_\Delta$, and $\underline{w}_\Delta(p_t, s_0) := \hat{w}_\Delta(p_t)$ if $t \geqslant \hat{t}_\Delta$. Similarly, let

$$\underline{w}_\Delta(p_t, s') := \begin{cases} \mathbb{E}_t\left[e^{-r(\hat{t}_\Delta - t)}\hat{w}_\Delta(p_{\hat{t}_\Delta})\right] & \text{if } t < \hat{t}_\Delta \\ \mathbb{E}_t\left[e^{-r\Delta}\hat{w}_\Delta(p_{t+\Delta})\right] & \text{if } t \geqslant \hat{t}_\Delta. \end{cases}$$

Note that $\underline{w}_\Delta(p_t, s') < \underline{w}_\Delta(p_t, s_0)$ for all $p_t > 0$.

Proposition 2. *The following strategy profile, $\mathcal{E}^*_\Delta$, constitutes an equilibrium:*

- $w_A(p_t, s'') = S(p_t)$ *and*

$$w_A(p_t, \{s_0, s'\}) = \begin{cases} 0 & \textit{if } t < \hat{t}_\Delta \\ \hat{w}_\Delta(p_t) & \textit{if } t \geqslant \hat{t}_\Delta. \end{cases}$$

- $w_B(p_t, s'') = S(p_t)$ *and*

$$w_B(p_t, w_A, s') = \begin{cases} \min\{w_A, S(p_t)\} & \text{if } t < t^*_\Delta \\ \max\{\underline{w}_\Delta(p_t, s'), \min\{w_A, S(p_t)\}\} & \text{if } t \geqslant t^*_\Delta. \end{cases}$$

-

$$d(p_t, s_0, w_A) = \begin{cases} A & \text{if } w_A \geqslant \underline{w}_\Delta(p_t, s_0) \\ \varnothing & \text{otherwise} \end{cases}$$

$$d(p_t, s', w_A, w_B) = \begin{cases} A & \text{if } w_A > w_B \geqslant \underline{w}_\Delta(p_t, s'),\ \text{or } w_A \geqslant \underline{w}_\Delta(p_t, s') > w_B \\ B & \text{if } w_B \geqslant \max\left\{\underline{w}_\Delta(p_t, s'), w_A\right\} \\ \varnothing & \text{otherwise} \end{cases}$$

$$d(p_t, s'', w_A, w_B) = \begin{cases} A & \text{if } w_A \geqslant w_B \geqslant V_\Delta(p_t) \\ B & \text{if } w_B > w_A \geqslant V_\Delta(p_t) \\ \varnothing & \text{otherwise.} \end{cases}$$

We henceforth refer to the equilibrum strategy profile $\mathcal{E}^*_\Delta$ in the statement of Proposition 2 as the efficient equilibrium. On the equilibrium path, each firm offers 0 until the cutoff time $\hat{t}_\Delta$, at which point each firm offers $\hat{w}_\Delta(p_t)$ when the state belongs to $\{s_0, s'\}$ and the entire surplus in state s''. Since $\mathcal{E}^*_\Delta$ coincides with $\mathcal{E}_\Delta$ whenever $p_t \geqslant \hat{p}_\Delta$, all that we need to prove is that neither firm has a profitable deviation for $p_t < \hat{p}_\Delta$. The difficulty is that, in $\mathcal{E}^*_\Delta$, the worker's reservation wage (in state s_0) is strictly less than $\hat{w}_\Delta(p_t)$ for all $p_t < \hat{p}_\Delta$ (which was the reservation wage in $\mathcal{E}_\Delta$). Consequently, the share of surplus which firms can extract prior to the cutoff time $\hat{t}_\Delta$ is higher in $\mathcal{E}^*_\Delta$ than in $\mathcal{E}_\Delta$. We show that, in spite of this remark, waiting until $\hat{t}_\Delta$ in order to hire the worker for a wage $\hat{w}_\Delta(\hat{p}_\Delta)$ yields each firm higher expected discounted payoff than offering the worker her reservation wage at any time before $\hat{t}_\Delta$.

6 Equilibrium Properties

In the previous section we constructed two equilibria. In both equilibria: (a) if the worker is hired by A, it is at time $\hat{t}_\Delta$ for a wage $\hat{w}(\hat{p}_\Delta)$; (b) following the observed arrival of the entrant before $\hat{t}_\Delta$, the worker is hired at time t^*_Δ for a wage $S(p^*_\Delta)$ (conditional on no slip-up occurring before then); (c) following the unobserved arrival of the entrant, the worker is hired at time $\hat{t}_\Delta$ at the latest. In the following theorem we show that these properties are common to *all* equilibria. The proofs for this section are in Appendix D.

Theorem 1. *In any equilibrium, conditional on no slip-up occurring before the cutoff time* $\hat{t}_\Delta$*:*

1. *firm A obtains a positive surplus if and only if firm B arrives after* $\hat{t}_\Delta$*;*
2. *if firm B arrives after* $\hat{t}_\Delta$*, firm A hires the worker at time* $\hat{t}_\Delta$ *for a wage* $\hat{w}_\Delta(\hat{p}_\Delta)$*;*
3. *if firm B arrives before* $\hat{t}_\Delta$ *and the entrant's arrival is unobserved, it hires the worker at some time* $t \leqslant \hat{t}_\Delta$.[21]

In view of Theorem 1, characterizing the cutoff time $\hat{t}_\Delta$ will enable us to quantify the extent of unraveling occurring in all equilibria. Our focus in the rest of this section is on the welfare implications of the theorem. First, all equilibria are payoff-equivalent for the incumbent since, no matter the equilibrium, A hires the worker at time $\hat{t}_\Delta$ and pays the wage $\hat{w}_\Delta(\hat{p}_\Delta)$ if $T_B > \hat{t}_\Delta$, and otherwise A does not obtain any surplus as it either competes a la Bertrand or loses the worker to the entrant. However, all equilibria are not payoff-equivalent for the worker, nor for the entrant. In equilibrium $\mathcal{E}_\Delta$ for instance, the worker is offered $\hat{w}_\Delta(p_t)$ or more for all t (as long as no slip-up has occured). This feature enables the worker to secure the wage $\hat{w}_\Delta(p_{T_B})$ at time T_B in case the entrant's arrival is unobserved and occurs before $\hat{t}_\Delta$. By contrast, in equilibrium $\mathcal{E}^*_\Delta$ the worker is forced to wait until $\hat{t}_\Delta$ in order to obtain $\hat{w}_\Delta(\hat{p}_\Delta)$. But the worker would strictly prefer accepting $\hat{w}_\Delta(p_{T_B})$ at time $T_B < \hat{t}_\Delta$ since all upside potential disappears the instant B's arrival is unobserved. Hence $\mathcal{E}_\Delta$ is better than $\mathcal{E}^*_\Delta$ from the worker's perspective. By the same token $\mathcal{E}^*_\Delta$ is better than $\mathcal{E}_\Delta$ from the viewpoint of firm B. To see this, note that whereas competition grows over time from the perspective of the incumbent, competition is constant from the perspective of the entrant. Consequently, following an unobserved arrival before $\hat{t}_\Delta$, firm B would prefer hiring the worker at the later time $\hat{t}_\Delta$ rather than immediately. Yet, as argued above, the worker's reservation wage drops the instant B's arrival is unobserved. This forces B to hire the worker as soon as offers from A are larger than the reservation wage in the state s'. In $\mathcal{E}^*_\Delta$ the latter effect forces B to hire the worker at time $\hat{t}_\Delta$; in $\mathcal{E}_\Delta$ it forces B to hire the worker at time $T_B < \hat{t}_\Delta$.

As it turns out, $\mathcal{E}^*_\Delta$ is firm B's (resp. the worker's) most-preferred (resp. least-preferred) equilibrium, while $\mathcal{E}_\Delta$ is firm B's (resp. the worker's) least-preferred (resp. most-preferred)

[21] At which exact time $t \leqslant \hat{t}_\Delta$ firm B hires the worker after an unobserved will depend on the equilibrium. For example, in equilibrium $\mathcal{E}_\Delta$ firm B hired the worker immediately after an unobserved arrival before $\hat{t}_\Delta$. In contrast, in equilibrium $\mathcal{E}^*_\Delta$, firm B waited until $\hat{t}_\Delta$ to hire the worker even if it arrived unobserved well before $\hat{t}_\Delta$.

equilibrium. Finally, observe that in $\mathcal{E}^*_\Delta$ the worker is never hired before the cutoff time $\hat{t}_\Delta$. It therefore follows from Theorem 1 that $\mathcal{E}^*_\Delta$ is the most efficient equilibrium. Theorem 2 summarizes all these results.

Theorem 2. *The set of equilibria satisfies the following properties:*

1. *all equilibria are payoff-equivalent for firm A,*
2. $\mathcal{E}^*_\Delta$ *is firm B's most-preferred equilibrium and the worker's least-preferred equilibrium,*
3. $\mathcal{E}_\Delta$ *is firm B's least-preferred equilibrium and the worker's most-preferred equilibrium,*
4. $\mathcal{E}^*_\Delta$ *is the most efficient equilibrium.*

Some additional remarks are worth highlighting. As the worker is never hired later in $\mathcal{E}_\Delta$ than in $\mathcal{E}^*_\Delta$ (and sometimes strictly earlier), notice that "employment" is higher in $\mathcal{E}_\Delta$ than in $\mathcal{E}^*_\Delta$. By the same token, the average talent of an employed worker is lower in $\mathcal{E}_\Delta$ than in $\mathcal{E}^*_\Delta$. Note too that in both equilibria talent and wages are non-uniform within and across firms. We elaborate on these observations in the next section.

7 Comparative Statics

In this section we present various comparative statics results. The proofs for this section are in Appendix E. For tractability and clarity of graphs, we concentrate on the limit as $\Delta \to 0$. To this end, define:

$$V(p) := \max_{T \geqslant 0} e^{-rT} \mathbb{E}_p \left[\max\{0, S(p_T)\} \right]. \tag{4}$$

Let t^* be the solution to (4) given intial belief p_0, and let p^* denote the belief at time t^* conditional on slip up. Then, whenever $p^* \in (0,1)$, it must satisfy the first-order condition

$$rS(p^*) = c(1-p^*)\eta.$$

Next, define $\hat{p}$ implicitly by

$$rS(\hat{p}) = c\eta(1-\hat{p}) - \lambda\Big[S(\hat{p}) - \Big((1-\beta)V(\hat{p}) + \beta\hat{w}(p)\Big)\Big], \tag{5}$$

as well as

$$\hat{w}(p) := \mathbb{E}\left[e^{-r\mathcal{T}} V(p_{\mathcal{T}})\right], \tag{6}$$

where $\mathcal{T}$ is exponentially distributed with intensity $(1-\beta)\lambda$.

We show in the appendix that $V(p) = \lim_{\Delta\to 0} V_\Delta(p)$, $\hat{w}(p) = \lim_{\Delta\to 0} \hat{w}_\Delta(p)$, $\hat{t} = \lim_{\Delta\to 0} \hat{t}_\Delta$, $t^* = \lim_{\Delta\to 0} t^*_\Delta$, $\hat{p} = \lim_{\Delta\to 0} \hat{p}_\Delta$, and $p^* := \lim_{\Delta\to 0} p^*_\Delta$. Consequently, all payoffs in this section correspond to limiting equilibrium payoffs as $\Delta \to 0$. Unless stated otherwise, our simulations use the following parameters: $\eta = b = c = \lambda = 1$, $\beta = 0.9$, $r = 0.1$, $p_0 = 0.3$.

7.1 Informational Frictions and Competition

We explore in this subsection the welfare effects of informational frictions (β) and of changing the competition (λ). Recall that by Theorem 1 all equilibria have the property that hiring is never delayed beyond $\hat{t}_\Delta$. Furthermore, in the efficient equilibrium, hiring is delayed until exactly $\hat{t}_\Delta$. Thus, $\hat{t}_\Delta$ represents the minimal amount of unraveling that could occur in any equilibrium. The following lemma describes the comparative statics of $\hat{t}_\Delta$ when Δ is sufficiently small as λ and β change.

Lemma 3. *The cutoff time $\hat{t}$ is decreasing in β and non-monotonic in λ. Moreover, $\hat{t} \to t^*$ whenever one of the following holds: (a) $\beta \to 0$, (b) $\lambda \to 0$, (c) $\lambda \to +\infty$ and $\beta > 0$.*[22]

The intuition behind Lemma 3 is as follows. We noted in the previous section that the cutoff belief $\hat{p}_\Delta$ balances the worker and firm A's collective gain from waiting to learn the worker's realized talent versus the loss from delaying hiring in case the worker is in fact talented. By reducing the worker and firm A's collective gain from waiting to learn the worker's realized talent, increasing β therefore lowers the cutoff belief $\hat{p}_\Delta$. The impact of λ is more complicated. On the one hand increasing λ raises the probability of firm B hiring the worker, which reduces the worker and firm A's joint incentive to wait. On the other hand, by Lemma 1, increasing λ augments $\hat{w}_\Delta(\cdot)$ and, thereby, raises the worker's payoff whenever firm B hires the worker in the state s'. The latter effect enhances the worker and firm A's gain from waiting to learn about talent. These countervailing forces suggest the non-monotonicity

[22]This lemma holds even for $\Delta > 0$ except for the statement about $\lambda \to +\infty$. When Δ is bounded away from zero, even as $\lambda \to +\infty$, $\hat{t}$ remains bounded away from t^*. The reason is in the way we've defined the discrete time game so that in each period a maximum of one offer arrives.

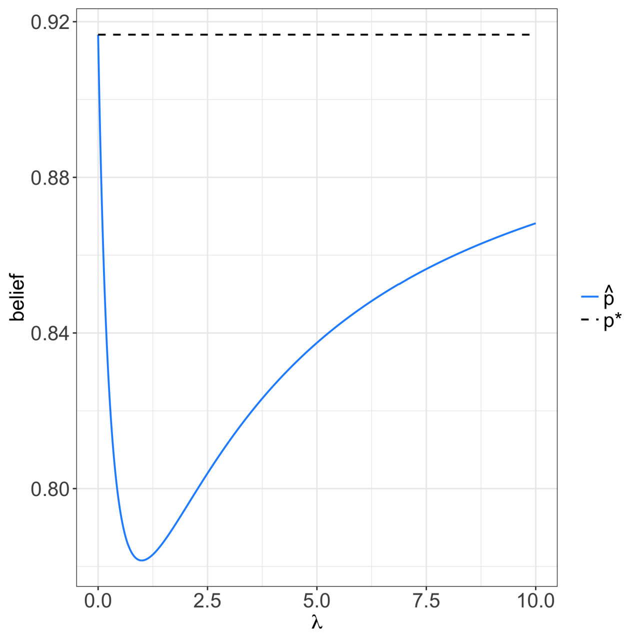

FIGURE 2

of $\hat{t}$ (and hence $\hat{p}$) in λ. Figure 2 illustrates $\hat{p}$ as a function of λ; simulations show that $\hat{p}$ is in fact quasi-convex in λ.

To highlight the intuition behind the limit results, note that when $\beta = 0$, i.e., absent informational frictions, the worker extracts all surplus after the arrival of the entrant. The entrant never receives any surplus. The worker and firm A therefore agree on a contract at the efficient time. Similarly, when $\lambda = 0$, the entire surplus is trivially captured by the coalition of firm A and the worker which leads to the efficient outcome. In contrast, if λ becomes arbitrarily large (and $\beta > 0$), the worker can threaten firm A with extracting the entire surplus after an observed arrival (which occurs at rate $(1-\beta)\lambda$). Thus, as this intensity increases to infinity, the worker can capture all surplus and conditional on no arrival will wait until exactly t^* to sign a contract with firm A.

The above lemma yields direct implications for social welfare, as seen in the next theorem. As it is the most efficient equilibrium (see Theorem 2) we focus henceforth on equilibrium $\mathcal{E}^*_\Delta$.[23] Let $\Pi^*_{\Delta i}$ denote player i's ex-ante expected discounted payoff in equilibrium $\mathcal{E}^*_\Delta$ and let W^*_Δ denote the ex-ante social welfare, that is, $W^*_\Delta = \Pi^*_{\Delta A} + \Pi^*_{\Delta B} + \Pi^*_{\Delta \text{worker}}$. As before, for tractability, we focus on the limit as $\Delta \to 0$ and so we additionally define $\Pi^*_i := \lim_{\Delta\to 0} \Pi^*_{\Delta i}, W^* := \lim_{\Delta \to 0} W^*_\Delta$.

[23] All of our results would be the same if we focused instead on equilibrium $\mathcal{E}_\Delta$.

Theorem 3. *(i)* W^* *is decreasing in* β*, and tends to* $V(p_0)$ *as* $\beta \to 0$*. (ii)* W^* *is non-monotonic in* λ*, and tends to* $V(p_0)$ *as* λ *tends to zero or infinity.*

The effect of β on social is welfare is straightforward, since increasing β lowers $\hat{t}$ (see Lemma 3) and simultaneously raises the relative probability with which the worker is hired at $\hat{t}$ rather than t^* (by increasing the probability that the worker is hired in the state s' rather than s''). The effect of λ is more subtle. As long as increasing λ raises $\hat{t}$ then increasing λ unambiguously improves social welfare. However, when increasing λ lowers $\hat{t}$, then social welfare can be negatively affected. We next analyze the players' equilibrium expected payoffs.

Proposition 3. $W^* > \Pi^*_{worker}$ *for all* λ *and* $W^* > \Pi^*_A$ *for all* $\lambda > 0$*. Moreover:*

1. Π^*_{worker} *is increasing in* λ*, decreasing in* β*, and* $W^* - \Pi^*_{worker} \to 0$ *as* $\lambda \to +\infty$*,*
2. Π^*_A *is decreasing in* λ*, increasing in* β*,* $W^* - \Pi^*_A \to 0$ *as* $\beta \to 0$*,*
3. Π^*_B *is non-monotonic in* λ*.*

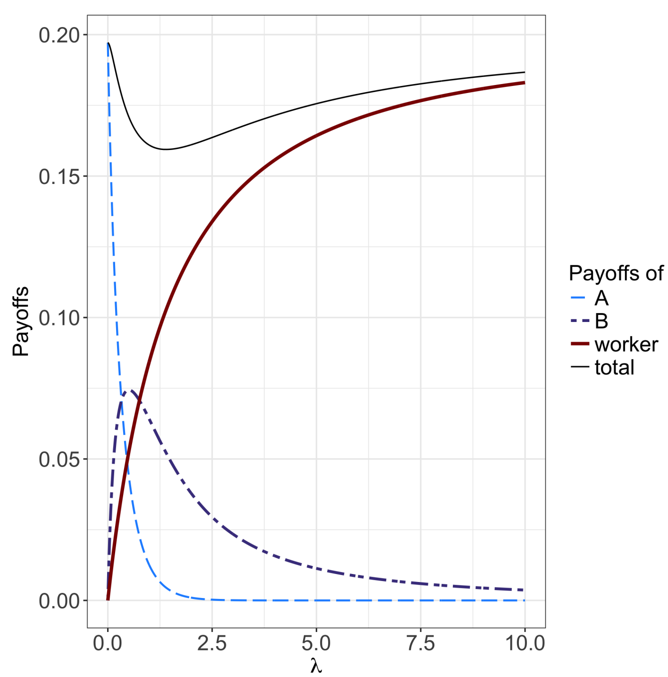

FIGURE 3

Figure 3 illustrates Π^*_i as a function of λ, and Figure 4 as a function of β, for $i \in \{A, B, \text{worker}\}$. Combining results from Theorem 3 and Proposition 3 highlights the existence of a fundamental trade-off between efficiency and inequality: fixing β and varying λ,

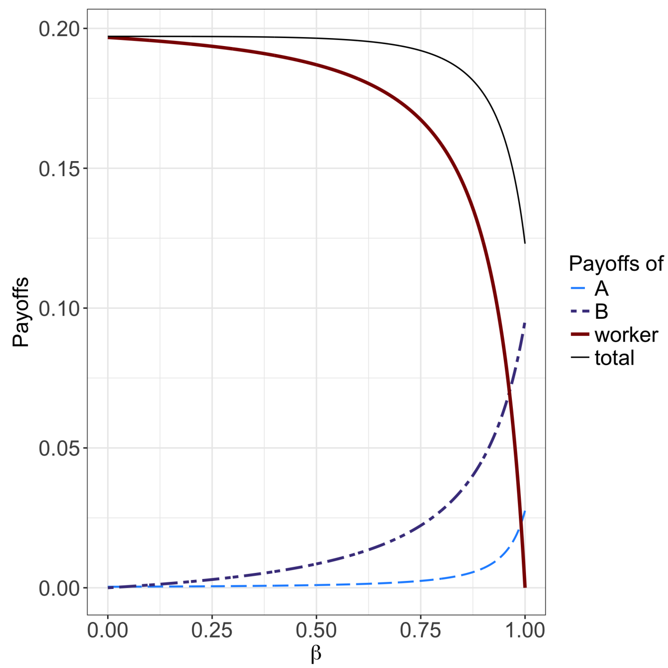

FIGURE 4

efficiency is attained if and only if either the worker or firm A appropriates the entire surplus. The logic of the trade-off is as follows. When bargaining power is distributed evenly between the worker and firm A (which occurs for intermediate values of λ) then B is able to claim a large part of the surplus. This in turn reduces the worker and firm A's collective gain from waiting to learn about talent, increasing unraveling.

The finding that raising λ can hurt firm B is explained by the fact that an increase in λ raises the worker's reservation wage in the state s'. Finally, the fact that increasing λ and decreasing β both worsen the welfare of A while improving the welfare of the worker should come as no surprise to the reader. Indeed the result is trivial in equilibrium $\mathcal{E}_\Delta$, since $\hat{w}_\Delta(\cdot)$ is decreasing in β and increasing in λ, and the worker is offered $\hat{w}_\Delta(p_t)$ at each point in time whenever the state belongs to $\{s_0, s'\}$ (conditional on no slip-up having occurred). Note however that establishing the result in equilibrium $\mathcal{E}^*_\Delta$ requires some care, since whenever the state belongs to $\{s_0, s'\}$ the worker is first offered $\hat{w}_\Delta(\cdot)$ at time $\hat{t}_\Delta$ and we saw in Lemma 3 that $\hat{t}_\Delta$ could increase with the parameter λ. The proof uses an envelope theorem argument.

7.2 Wages and Average Talent

Next, we examine the distributions of talent and wages within and across firms. As $p^*_\Delta < 1$ and since the worker is never hired passed the belief p^*_Δ, each firm exhibits a mixture of good and bad employees (in all equilibria). However, since firm A and firm B hire at different times, the respective shares of good and bad employees vary across the two firms.

So far, we assumed for expository purposes that (in equilibrium) firm A always hires the worker in case of an observed arrival. There is also a pure strategy equilibrium in which firm B always hires the worker. Payoffs are the same in those two equilibria, but the average talent hired by each firm is affected. For this reason we allow in this subsection for a mixed action in the case of an observed arrival, in which A and B hire the worker with probability $\frac{1}{2}$ each. The focus, as before, is on the efficient equilibrium $\mathcal{E}^*_\Delta$.

As talent is a binary variable, it is sufficient to examine average talent. At firm A, average talent is given by

$$\text{Average talent at A} = \frac{(1-e^{-\lambda\hat{t}})\frac{1}{2}(1-\beta)\frac{\hat{p}}{p^*}}{(1-e^{-\lambda\hat{t}})\frac{1}{2}(1-\beta)\frac{\hat{p}}{p^*}+e^{-\lambda\hat{t}}}p^* + \frac{e^{-\lambda\hat{t}}}{(1-e^{-\lambda\hat{t}})\frac{1}{2}(1-\beta)\frac{\hat{p}}{p^*}+e^{-\lambda\hat{t}}}\hat{p}.$$

To see this, note that the probability that $T_B \leqslant \hat{t}$ is $1-e^{-\lambda\hat{t}}$, in which case firm A hires the worker at the belief p^* with probability $\frac{1}{2}(1-\beta)$; starting from $\hat{p}$, the probability that the worker does not slip up before hitting p^* is $\frac{\hat{p}}{p^*}$. If $T_B > \hat{t}$ then firm A hires the worker at the belief $\hat{p}$. We find in a similar way that, at firm B, average talent is given by

$$\text{Average talent at B} = \frac{\frac{1}{2}(1-\beta)\frac{\hat{p}}{p^*}}{\frac{1}{2}(1-\beta)\frac{\hat{p}}{p^*}+\beta}p^* + \frac{\beta}{\frac{1}{2}(1-\beta)\frac{\hat{p}}{p^*}+\beta}\hat{p}.$$

This implies among other things that average talent is greater at A than at B if and only if

$$\beta > \frac{e^{-\lambda\hat{t}}}{1-e^{-\lambda\hat{t}}}.$$

The intuition behind the latter condition is as follows. Increasing β changes the composition of workers hired by B, by reducing the fraction of workers hired at the belief p^* and increasing the fraction of workers hired early at the belief $\hat{p}$. By contrast, increasing λ leaves the composition of workers hired by B unaffected, but transforms the composition of workers hired by A, by reducing the fraction of workers hired early at the belief $\hat{p}$ and increasing the fraction of

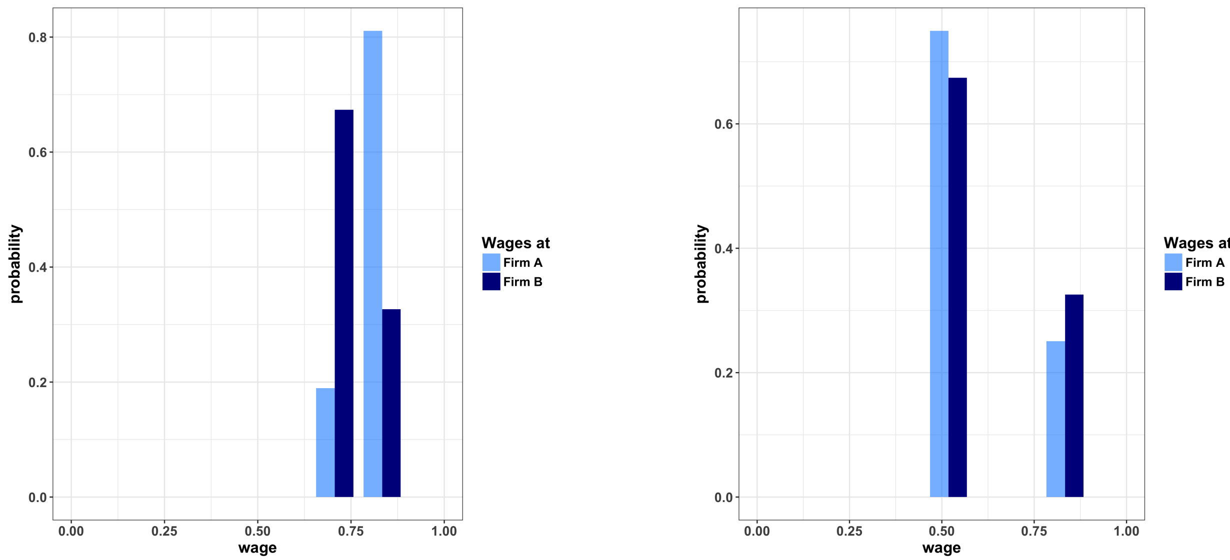

FIGURE 5

workers hired at the belief p^*. The distribution of wages within and across firms is illustrated in Figure 5, for two values of λ. On the left panel, $\lambda = 1$, and on the right panel $\lambda = 0.3$. The light blue bars correspond to firm A, and the dark blue bars correspond to firm B. The fractions of high and low wages at firm B is the same in both panels. However, as λ goes from 0.3 to 1, the fraction of high wages goes from 25% to 81%.

7.3 The Speed of Learning

Lastly, we discuss briefly the effect of η, representing the speed of learning about the worker's talent. Observe first that the planner's value function, $V_\Delta(p)$, is increasing in η.[24] By (2), it ensues that $\hat{w}_\Delta(p)$ is also increasing in η, from which we conclude, using (5), that $\hat{p}$ increases with η. Figure 6 illustrates the payoffs as a function of η. Both the worker and firm A benefit from faster learning. However, increasing η can hurt firm B. This follows from Theorem 1 and the fact that $\hat{t}$ may decrease with η, leaving less time for the entrant to arrive before the worker is hired by the incumbent. At the same time, firm B can benefit from a high speed of learning due to the fact that, since $\hat{p}$ increases with η, $S(\hat{p}) - \hat{w}(\hat{p})$ must increase with η as well (by part 4 of Lemma 1).

[24] If $\eta_2 > \eta_1$ and the social planner chooses the cutoff belief p_1^* then this belief is reached sooner. That is, the planner is better off facing η_2 than η_1 even if he does not adjust his strategy.

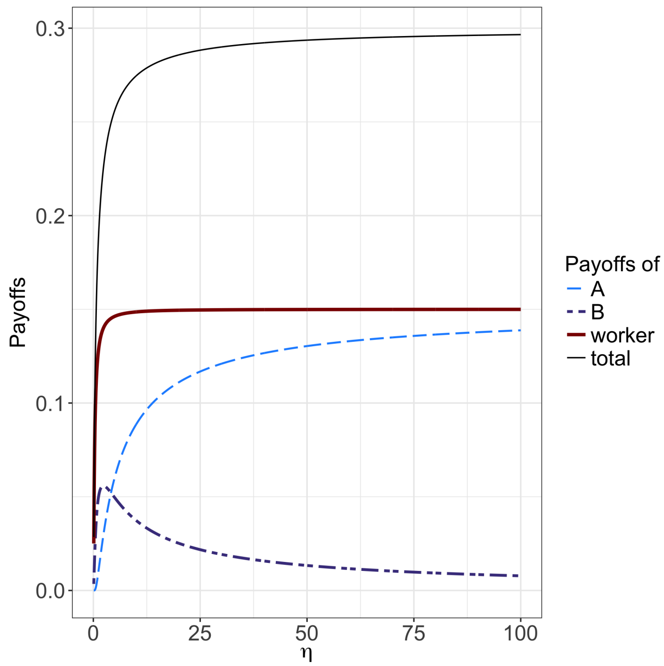

FIGURE 6

8 Conclusion

The principal contribution of our paper is to shed light on the link between wage bargaining and unraveling. We show that informational frictions related to bargaining play a key role in the extent of unraveling occurring in equilibrium, and that competition alone is not enough to cause unraveling. Surprisingly, in the presence of informational frictions, the relationship between competition and unraveling is non-monotonic. Increasing competition at first increases unraveling, but then decreases unraveling passed a certain point. We also highlight a novel trade-off between unraveling and inequality: the more equal the division of surplus between firms and workers, the greater the extent of unraveling.

In order to achieve tractability, we make several modelling assumptions. The bargaining protocol of our model is very simple, and can only partly capture the complexity of bargaining occurring in the real world. We also assume that firms are completely symmetric in terms of payoffs. This allows us to capture the trade-offs that have not been captured by the unraveling literature that is mostly concerned with match-specific productivity. Worker-specific productivity ("talent") is particularly important in thin speciality markets where we observe a lot of unraveling. Finally, we assume that contracts are signed once and for all. This is to some extent justified by firms' efforts to increase switching costs, by e.g., offering subsidies for homes. Similarly, firms face restrictions in how fast they can fire workers. Nevertheless, it would be interesting to think about a richer class of contracts in a similar setup.

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A Preliminaries: Single-Crossing Conditions

Before we present the proofs of the paper, we prove a single-crossing result for a class of value functions that are relevant in many proofs. The following lemma provides sufficient conditions under which such a value function satisfies a single-crossing condition.

To this end, let us define a class of continuations values given a function $g(p) \geqslant 0$ by:

$$W_\Delta(p_t, g) := e^{-r\Delta}\mathbb{E}_t\left[e^{-\lambda\Delta}\max\{0, S(p_{t+\Delta})\} + \left(1 - e^{-\lambda\Delta}\right)(\beta g(p_{t+\Delta}) + (1-\beta)V_\Delta(p_{t+\Delta}))\right].$$

Consider a decision maker who can either take $S(p_t)$ today or wait for a period and obtain $W_\Delta(p_t, g)$. Then, the following lemma provides sufficient conditions under which there is a unique optimal stopping time $\bar{t}_\Delta$ at which the decision maker should take $S(p_t)$.

Lemma 4. *Suppose that $g : (0,1) \to [0, +\infty)$ with $g(0) = 0$ satisfies the following inequalities for all t: $\mathbb{E}_t\left[e^{-r\Delta}g(p_{t+\Delta})\right] \leqslant g(p_t) < V_\Delta(p_t)$. Then there exists $\bar{t}_\Delta$ such that for all t,*

$$\begin{cases} 0 < W_\Delta(p_t, g) - S(p_t) & \text{if } t < \bar{t}_\Delta, \\ 0 > W_\Delta(p_t, g) - S(p_t) & \text{if } t \geqslant \bar{t}_\Delta.^{25} \end{cases}$$

Proof: Let $M(p_t) := W_\Delta(p_t, g) - S(p)$ denote the marginal benefit to waiting an additional period. This can be decomposed into two terms, $M(p_t) = X(p_t) - Y(p_t)$ where:

$$X(p_t) = \mathbb{E}_t\left[e^{-r\Delta}\max\{0, S(p_{t+\Delta})\}\right] - S(p_t),$$
$$Y(p_t) = \mathbb{E}_t\left[e^{-r\Delta}\left(1 - e^{-\lambda\Delta}\right)(\max\{0, S(p_{t+\Delta})\} - \beta g(p_{t+\Delta}) - (1-\beta)V_\Delta(p_{t+\Delta}))\right].$$

First note that for all $t \geqslant t^*_\Delta$, $Y(p_t) > 0 \geqslant X(p_t)$ which then implies that $M(p_t) < 0$. Thus for the theorem, it is sufficient to show that if $t < t^*_\Delta$ and $M(p_t) \leqslant 0$, then $M(p_{t+\Delta}) < 0$. Note that if $t + \Delta \geqslant t^*_\Delta$, we are already done. Thus let us assume that $t < t + \Delta < t^*_\Delta$. Because $t + \Delta < t^*_\Delta$, we have:

$$S(p_{t+\Delta}) < \mathbb{E}_{t+\Delta}\left[e^{-r\Delta}\max\{0, S(p_{t+2\Delta})\}\right],$$
$$g(p_{t+\Delta}) \geqslant \mathbb{E}_{t+\Delta}\left[e^{-r\Delta}g(p_{t+2\Delta})\right],$$
$$V_\Delta(p_{t+\Delta}) = \mathbb{E}_{t+\Delta}\left[e^{-r\Delta}V_\Delta(p_{t+2\Delta})\right].$$

[25] We ignore the case in which equality holds at exactly $\bar{t}_\Delta$ since this occurs only in knife-edge cases.

Combining the above inequalities, we obtain:

$$(1 - e^{-\lambda\Delta})\mathbb{E}_{t+\Delta}\left[\max\{0, S(p_{t+\Delta})\} - \beta g(p_{t+\Delta}) - (1-\beta)V_\Delta(p_{t+\Delta})\right] < Y(p_{t+\Delta}).$$

Since we assumed that $M(p_t) \leqslant 0$, $0 < X(p_t) \leqslant Y(p_t)$, which means that

$$\mathbb{E}_t\left[\max\{0, S(p_{t+\Delta})\} - \beta g(p_{t+\Delta}) - (1-\beta)V_\Delta(p_{t+\Delta})\right] > 0.$$

This implies that

$$Y(p_t) < (1 - e^{-\lambda\Delta})\mathbb{E}_t\left[\max\{0, S(p_{t+\Delta})\} - \beta g(p_{t+\Delta}) - (1-\beta)V_\Delta(p_{t+\Delta})\right] < Y(p_{t+\Delta}).$$

Furthermore as we will see in Section B, we know that $X(p_{t+\Delta}) < X(p_t)$. Combining these inequalities, we obtain:

$$M(p_{t+\Delta}) = X(p_{t+\Delta}) - Y(p_{t+\Delta}) < X(p_t) - Y(p_t) = M(p_t) \leqslant 0.$$

■

B Proofs of Section 4

We first provide details of the derivations concerning the social planner's problem,

$$V_\Delta(p) := \max_{t \in \mathbb{T}_\Delta} \mathbb{E}_p\left[e^{-rt}\max\{0, S(p_t)\}\right].$$

Consider the marginal benefit of waiting an extra period. Using $S(p_t) = (b+c)p_t - c$, we obtain

$$\begin{aligned}
\mathbb{E}_t\left[e^{-r\Delta}\max\{0, S(p_{t+\Delta})\}\right] - S(p_t) &= e^{-r\Delta}\left(p_t + (1-p_t)e^{-\eta\Delta}\right)\left((b+c)p_{t+\Delta} - c\right) - S(p_t) \\
&= e^{-r\Delta}\left((b+c)p_t - cp_t - c(1-p_t)e^{-\eta\Delta}\right) - S(p_t) \\
&= -(b+c)p_t(1 - e^{-r\Delta}) + c[1 - e^{-r\Delta}\left(p_t + (1-p_t)e^{-\eta\Delta}\right)].
\end{aligned}$$

The right-hand side in the last equation is strictly decreasing in t. Thus, there exists some t^*_Δ such that

$$\begin{cases} \mathbb{E}_t\left[e^{-r\Delta}\max\{0, S(p_{t+\Delta})\}\right] - S(p_t) > 0 & \text{if } t < t^*_\Delta \\ \mathbb{E}_t\left[e^{-r\Delta}\max\{0, S(p_{t+\Delta})\}\right] - S(p_t) < 0 & \text{if } t \geqslant t^*_\Delta. \end{cases}$$

Furthermore, given the above, it is straightforward to show that indeed such a t^*_Δ is the unique maximizer of the social planner's problem.

Proof of Lemma 1: We prove below each of the properties listed in the lemma:

1. We have

$$V_\Delta(p) = \sup_{T\in\mathbb{T}_\Delta} \mathbb{E}_p\left[e^{-rT}\max\{0, S(p_T)\}\right] = \sup_{T\in\mathbb{T}_\Delta} \mathbb{E}_p[e^{-rT}V_\Delta(p_T)] > \mathbb{E}_p[e^{-r\hat{T}}V_\Delta(p_{\hat{T}})] = \hat{w}_\Delta(p).$$

2. $V_\Delta(p)$ is increasing in p and, for any t, p_t is increasing in the initial belief p_0. Thus, $\hat{w}_\Delta(p)$ given by (2) is increasing in p.

3. Immediate from (2).

4. First note that for all $\tau \geqslant t^*_\Delta$. $S(p_\tau) = V_\Delta(p_\tau)$ and therefore, $S(p_t) > \hat{w}_\Delta(p_t)$ for all $t \geqslant t^*_\Delta$. Thus to prove the claim, it suffices to show that if $S(p_{t+\Delta}) - \hat{w}_\Delta(p_{t+\Delta}) < 0$ at time $t+\Delta$ then $S(p_t) - \hat{w}_\Delta(p_t) < 0$ for $t < t+\Delta < t^*_\Delta$. If $S(p_{t+\Delta}) - \hat{w}_\Delta(p_{t+\Delta}) < 0$ and $p_{t+\Delta} \neq 0$, then

$$S(p_t) < e^{-r\Delta}\left(p_t + (1-p_t)e^{-\eta\Delta}\right) S(p_{t+\Delta})$$

and

$$\begin{aligned} \hat{w}(p_t) &= e^{-r\Delta}\left(p_t + (1-p_t)e^{-\eta\Delta}\right)\left[\left(e^{-\lambda\Delta} + \left(1-e^{-\lambda\Delta}\right)\beta\right)\hat{w}_\Delta(p_{t+\Delta}) + (1-\beta)\left(1-e^{-\lambda\Delta}\right)V_\Delta(p_{t+\Delta})\right]) \\ &> e^{-r\Delta}\left(p_t + (1-p_t)e^{-\eta\Delta}\right)\hat{w}_\Delta(p_{t+\Delta}). \end{aligned}$$

Thus,

$$S(p_t) - \hat{w}_\Delta(p_t) < e^{-r\Delta}\left(p_t + (1-p_t)e^{-\eta\Delta}\right)\left(S(p_{t+\Delta}) - \hat{w}_\Delta(p_{t+\Delta})\right) < 0.$$

■

Proof of Lemma 2: By (2), we have

$$\hat{w}_\Delta(p_t) = e^{-r\Delta}\mathbb{E}_t\left[\left(e^{-\lambda\Delta} + \left(1-e^{-\lambda\Delta}\right)\beta\right)\hat{w}_\Delta(p_{t+\Delta}) + (1-\beta)\left(1-e^{-\lambda\Delta}\right)V_\Delta(p_{t+\Delta})\right]. \quad (7)$$

Repeated applications of (7) yield, for any $k = 1, 2, \ldots,$

$$\begin{aligned}\hat{w}_\Delta(p_t) &= \mathbb{E}_t\left[\sum_{n=0}^{k} e^{-r\Delta(n+1)}e^{-\lambda\Delta n}(1-e^{-\lambda\Delta})\big[(1-\beta)V_\Delta(p_{t+n\Delta}) + \beta\hat{w}(p_{t+n\Delta})\big]\right. \\ &\left. \quad + e^{-(r+\lambda)\Delta k}\hat{w}(p_{t+k\Delta})\right].\end{aligned}$$

As $k \to \infty$ this is equivalent to (3). ■

C Proofs of Section 5

Before proving Proposition 1, we prove an auxiliary corollary of Lemma 4 with $g = \hat{w}_\Delta$.

Corollary 1. *There exists some $\hat{t}_\Delta \geqslant 0$ such that*

$$\begin{cases} \mathbb{E}_t\left[e^{-(r+\lambda)\Delta}\max\{0, S(p_{t+\Delta}) - \hat{w}_\Delta(p_{t+\Delta})\}\right] > S(p_t) - \hat{w}_\Delta(p_t) & \text{if } t < \hat{t}_\Delta \\ \mathbb{E}_t\left[e^{-(r+\lambda)\Delta}\max\{0, S(p_{t+\Delta}) - \hat{w}_\Delta(p_{t+\Delta})\}\right] < S(p_t) - \hat{w}_\Delta(p_t) & \text{if } t \geqslant \hat{t}_\Delta. \end{cases}$$

Consequently, for all t,

$$\max\left\{\hat{t}_\Delta, t\right\} = \arg\max_{\tau \geqslant t} \mathbb{E}_t\left[e^{-(r+\lambda)(\tau-t)}\max\{0, S(p_\tau) - \hat{w}_\Delta(p_\tau)\}\right].$$

Proof: Note that $\hat{w}(0) = 0$ and that

$$\begin{aligned}&\mathbb{E}_t\left[e^{-r\Delta}\hat{w}_\Delta(p_{t+\Delta})\right] \\ &\leqslant e^{-r\Delta}\mathbb{E}_t\left[\left(1-(1-\beta)\left(1-e^{-\lambda\Delta}\right)\right)\hat{w}_\Delta(p_{t+\Delta}) + (1-\beta)\left(1-e^{-\lambda\Delta}\right)V_\Delta(p_{t+\Delta})\right] = \hat{w}_\Delta(p_t).\end{aligned}$$

Furthermore we already know from Lemma 1 that $\hat{w}_\Delta(p_t) \leqslant V_\Delta(p_t)$. Thus by substituting $g = \hat{w}_\Delta$, we can apply Lemma 4 to conclude that there exists some $\hat{t}_\Delta$ such that

$$\begin{cases} S(p_t) < W_\Delta(p_t, \hat{w}_\Delta) & \text{if } t < \hat{t}_\Delta, \\ S(p_t) > W_\Delta(p_t, \hat{w}_\Delta) & \text{if } t \geqslant \hat{t}_\Delta. \end{cases}$$

Subtracting through by $\hat{w}_\Delta(p_t)$ we obtain:

$$\begin{cases} S(p_t) - \hat{w}_\Delta(p_t) < W_\Delta(p_t, \hat{w}_\Delta) - \hat{w}_\Delta(p_t) & \text{if } t < \hat{t}_\Delta, \\ S(p_t) - \hat{w}_\Delta(p_t) > W_\Delta(p_t, \hat{w}_\Delta) - \hat{w}_\Delta(p_t) & \text{if } t \geqslant \hat{t}_\Delta. \end{cases}$$

But note that

$$W_\Delta(p_t, \hat{w}_\Delta) - \hat{w}_\Delta(p_t) = e^{-(r+\lambda)\Delta}\mathbb{E}_t\left[\max\{0, S(p_{t+\Delta}) - \hat{w}_\Delta(p_{t+\Delta})\}\right].$$

As a result, we have:

$$\begin{cases} S(p_t) - \hat{w}_\Delta(p_t) < e^{-(r+\lambda)\Delta}\mathbb{E}_t\left[e^{-(r+\lambda)\Delta}\max\{0, S(p_{t+\Delta}) - \hat{w}_\Delta(p_{t+\Delta})\}\right] & \text{if } t < \hat{t}_\Delta, \\ S(p_t) - \hat{w}_\Delta(p_t) > e^{-(r+\lambda)\Delta}\mathbb{E}_t\left[e^{-(r+\lambda)\Delta}\max\{0, S(p_{t+\Delta}) - \hat{w}_\Delta(p_{t+\Delta})\}\right] & \text{if } t \geqslant \hat{t}_\Delta. \end{cases}$$

Then it is clear that

$$\max\left\{\hat{t}_\Delta, t\right\} = \arg\max_{\tau \geqslant t} \mathbb{E}_t\left[e^{-(r+\lambda)\Delta}\max\{0, S(p_\tau) - \hat{w}_\Delta(p_\tau)\}\right].$$

■

Proof of Proposition 1: We prove that $\mathcal{E}_\Delta$ is an equilibrium. We first show that the worker's strategy is optimal given firms' strategies.

1. $\underline{s_t = s''}$: When $s_t = s''$ the worker can guarantee itself $V_\Delta(p_t)$ given both firms offer $S(p_t)$ in all future periods. This implies that the worker's best response is to accept $\max\{w_A, w_B\}$ if and only if $\max\{w_A, w_B\} \geqslant V_\Delta(p_t)$.

2. $\underline{s_t = s'}$: First note that for any $\tau > t$,

$$\begin{aligned} \mathbb{E}_t\left[e^{-r(\tau-t)}\hat{w}_\Delta(p_\tau)\right] &= \mathbb{E}_t\left[e^{-r(\tau-\Delta-t)}\mathbb{E}_{\tau-\Delta}\left[e^{-r\Delta}\hat{w}_\Delta(p_\tau)\right]\right] \\ &= \mathbb{E}_t\left[e^{-r(\tau-\Delta-t)}\underline{w}_\Delta(p_{\tau-\Delta}, s')\right] \\ &< \mathbb{E}_t\left[e^{-r(\tau-\Delta-t)}\hat{w}_\Delta(p_{\tau-\Delta})\right] < \cdots < \underline{w}_\Delta(p_t, s'). \end{aligned}$$

   Therefore,

$$\underline{w}_\Delta(p_t, s') = \max_{\tau > t} e^{-r(\tau-t)}\left(p_t + (1-p_t)e^{-\eta(\tau-t)}\right)\hat{w}_\Delta(p_\tau).$$

   Since in state s', all offers at times $\tau = t+1, t+2, \ldots$ are $\hat{w}_\Delta(p_\tau)$ by both firms, the best payoff that the worker can obtain from waiting at least one period is indeed $\underline{w}_\Delta(p_t, s')$. As a result, the optimal strategy of the worker is to accept $\max\{w_A, w_B\}$ if and only if $\max\{w_A, w_B\} \geqslant \underline{w}_\Delta(p_t, s')$.

3. $\underline{s_t = s_0}$: Note that after the state changes to $s_\tau = s'$, the worker's continuation value at time τ is $\hat{w}_\Delta(p_\tau)$. Similarly, if the state changes to $s_\tau = s''$ at time τ, then the worker's continuation value at time τ is $V_\Delta(p_\tau)$. By the construction of $\hat{w}_\Delta$, given any stopping time τ such that $\tau \geqslant t+\Delta$ almost surely that is measurable with respect to the worker's information, note that

$$\hat{w}_\Delta(p_t) = \mathbb{E}_t\left[\mathbf{1}(\tau < T_1)e^{-r\tau}\hat{w}_\Delta(p_\tau) + \mathbf{1}(\tau \geqslant T_1)e^{-rT_1}\left(\beta\hat{w}_\Delta(p_{T_1}) + (1-\beta)V_\Delta(p_{T_1})\right)\right]$$

where T_1 is a random time that is geometrically distributed with success rate $(1-e^{-\lambda\Delta})$. As a result, $\hat{w}_\Delta(p_t)$ is exactly the maximum continuation value that the worker can obtain from rejecting an offer at time t and waiting for the optimal time to accept. Thus the worker's best response is indeed the worker strategy specified by $\mathcal{E}_\Delta$.

Next we show that firm A's strategy is a best response to firm B and worker's strategies:

1. $\underline{s_t = s''}$: Here B offers $S(p_t)$. Thus A cannot achieve strictly positive expected payoff and offering $S(p_t)$ is a weak best response.[26]

2. $\underline{s_t \in \{s_0, s'\}}$: Observe first that if $s_t = s'$ the only way A can hire the worker is by offering more than the total surplus because B always matches its offer. Hence if $s_t \in \{s_0, s'\}$, firm A maximizes its payoff conditional on $s_t = s_0$. By Corollary 1,

$$S(p_t) - \hat{w}_\Delta(p_t) < e^{-(r+\lambda)(\hat{t}_\Delta - t)}\mathbb{E}_t\left[\max\{0, S(p_{\hat{t}_\Delta}) - \hat{w}_\Delta(p_{\hat{t}_\Delta})\}\right] \text{ for all } t < \hat{t}_\Delta, \tag{8}$$
$$S(p_t) - \hat{w}_\Delta(p_t) > e^{-(r+\lambda)(s-t)}\mathbb{E}_t\left[\max\{0, S(p_s) - \hat{w}_\Delta(p_s)\}\right] \text{ for all } s > t \geqslant \hat{t}_\Delta. \tag{9}$$

Suppose first that $t < \hat{t}_\Delta$. In this case, by playing the proposed equilibrium strategy, firm A obtains a payoff of

$$e^{-(r+\lambda)(\hat{t}_\Delta - t)}\mathbb{E}_t\left[\max\left\{0, S(p_{\hat{t}_\Delta}) - \hat{w}_\Delta(p_{\hat{t}_\Delta})\right\}\right].$$

A one-stage deviation of offering $w_A < \hat{w}_\Delta(p_t)$ gives exactly the same payoff while a

[26]Recall that in state $s_t = s''$, both firms simultaneously make an offer.

one-stage deviation of $w_A > \hat{w}_\Delta(p_t)$ gives a payoff of

$$S(p_t) - w_A < S(p_t) - \hat{w}_\Delta(p_t) < e^{-(r+\lambda)(\hat{t}_\Delta - t)} \mathbb{E}_t \left[\max\{0, S(p_{\hat{t}_\Delta}) - \hat{w}_\Delta(p_{\hat{t}_\Delta})\}\right]$$

where the second inequality follows from (8). Thus the offer of $\hat{w}_\Delta(p_t)$ is optimal for firm A.

Suppose next that $t \geqslant \hat{t}_\Delta$. Then note that by playing its equilibrium strategy, firm A receives a payoff of $S(p_t) - \hat{w}_\Delta(p_t)$. On the other hand, offering $w_A > \hat{w}_\Delta(p_t)$ is clearly suboptimal, while offering $w_A < \hat{w}_\Delta(p_t)$ leads to a rejection today, yielding a payoff of at most

$$\max_{s>t} e^{-(r+\lambda)(s-t)} \mathbb{E}_t \left[\max\left\{0, S(p_s) - \hat{w}_\Delta(p_s)\right\}\right] < S(p_t) - \hat{w}_\Delta(p_t),$$

where the inequality follows from (9). Thus we conclude that firm A's strategy is indeed a best response.

Lastly, we show that B's strategy is a best response to the other players' strategies.

1. $\underline{s_t = s''}$: The arguments are the same that we used for firm A.

2. $\underline{s_t = s'}$: Suppose that firm A has made an offer of w_A at time t. If $w_A \geqslant \underline{w}_\Delta(p_t, s')$, it is clear that the best response is to offer $\min\{S(p_t), w_A\}$. Suppose on the contrary that $w_A < \underline{w}_\Delta(p_t, s')$. Assume first that $t < t^*_\Delta$. Offering any wage $w_B < \underline{w}_\Delta(p_t, s')$ leads to a payoff of $\mathbb{E}_t \left[e^{-r\Delta}(\max\{0, S(p_{t+\Delta})\} - \hat{w}_\Delta(p_{t+\Delta}))\right]$. By offering $w_B \geqslant \underline{w}_\Delta(p_t, s')$, leads to a payoff of

$$\begin{aligned} S(p_t) - w_B \leqslant S(p_t) - \underline{w}_\Delta(p_t, s') &= S(p_t) - \mathbb{E}_t \left[e^{-r\Delta} \hat{w}_\Delta(p_{t+\Delta})\right] \\ &\leqslant \mathbb{E}_t \left[e^{-r\Delta} \max\{0, S(p_{t+\Delta}) - \hat{w}_\Delta(p_{t+\Delta})\}\right]. \end{aligned}$$

   Thus in this scenario, it is a best response for firm B to offer $w_B = w_A$.

   Finally suppose that $t \geqslant t^*_\Delta$. If $w_A < \underline{w}_\Delta(p_t, s')$, then offering the wage $\underline{w}_\Delta(p_t, s')$ leads to payoff of $S(p_t) - \underline{w}_\Delta(p_t, s')$. Clearly offering $w_B > \underline{w}_\Delta(p_t, s')$ is suboptimal while offering $w_B < \underline{w}_\Delta(p_t, s')$ leads to a payoff of

$$\mathbb{E}_t \left[e^{-r\Delta} \max\{0, S(p_{t+\Delta}) - \hat{w}_\Delta(p_{t+\Delta})\}\right] < S(p_t) - \underline{w}_\Delta(p_t, s').$$

   Thus the offer of $\underline{w}_\Delta(p_t, s')$ is a best response.

■

Proof of Proposition 2: Since $\mathcal{E}^*_\Delta$ coincides with $\mathcal{E}_\Delta$ at all times $t \geqslant \hat{t}_\Delta$, it remains to show incentive compatibility of strategies at times $t < \hat{t}_\Delta$. Thus throughout the remainder of the proof, we will restrict attention to $t < \hat{t}_\Delta \leqslant t^*_\Delta$. Let us first define the following:

$$\overline{w}_\Delta(p_t, s') = \begin{cases} \underline{w}_\Delta(p_t, s') & \text{if } t < \hat{t}_\Delta \\ \hat{w}_\Delta(p_t) & \text{if } t \geqslant \hat{t}_\Delta, \end{cases}$$

and $\overline{w}_\Delta(0, s') = 0$. Note that $\overline{w}_\Delta(p_t, s')$ represents the continuation value to the worker at time t conditional on the state s'. At all times $t \geqslant \hat{t}_\Delta$, $\overline{w}_\Delta(p_t, s') > \underline{w}_\Delta(p_t, s')$ since taking an offer of $\hat{w}_\Delta(p_t)$ at time t is strictly better than waiting until time $t + \Delta$ to take the offer of $\hat{w}_\Delta(p_{t+\Delta})$.

We first show that the worker's strategy is optimal given firms' strategies.

1. $\underline{s_t = s''}$: When $s_t = s''$, the worker can guarantee itself $V_\Delta(p_t)$ given that both firms offer $S(p_t)$ in all future periods. This implies that the worker's best response is to accept $\max\{w_A, w_B\}$ if and only if $\max\{w_A, w_B\} \geqslant V_\Delta(p_t)$.

2. $\underline{s_t = s'}$: At all times $\tau < \hat{t}_\Delta$, offers of both firms are 0 in state s'. Thus, the most that the worker obtain to rejecting the time t offer is

$$\mathbb{E}_t\left[e^{-r(\hat{t}_\Delta - t)}\hat{w}(p_{\hat{t}_\Delta})\right] = \underline{w}_\Delta(p_t, s').$$

   Thus, the worker's optimal strategy is to accept $\max\{w_A, w_B\}$ if and only if $\max\{w_A, w_B\} \geqslant \underline{w}_\Delta(p_t, s')$.

3. $\underline{s_t = s_0}$: Note that if the state changes to s' at time $\tau < \hat{t}_\Delta$, then the worker's continuation payoff is $\overline{w}_\Delta(p_t, s')$. Similarly, if the state changes to s'' at time $\tau < \hat{t}_\Delta$, then the worker's continuation payoff at time τ is $V_\Delta(p_t)$. As a result, the worker's continuation payoff to waiting a period and then playing according to the equilibrium strategy is given by:

$$e^{-r(\hat{t}_\Delta - t)}\mathbb{E}_t\left[\left(1-(1-\beta)\left(1-e^{-\lambda(\hat{t}_\Delta - t)}\right)\right)\hat{w}_\Delta(\hat{p}_\Delta) + (1-\beta)\left(1-e^{-\lambda(\hat{t}_\Delta - t)}\right)V_\Delta(\hat{p}_\Delta)\right],$$

   which is exactly $\underline{w}_\Delta(p_t, s_0)$. Thus, the worker's optimal strategy is to accept w_A if and only if $w_A \geqslant \underline{w}_\Delta(p_t, s_0)$.

We now show that firm A's strategy is optimal given the worker and firm B's strategies.

1. $\underline{s_t = s''}$: Here B offers $S(p_t)$. Thus A cannot achieve strictly positive expected payoff and offering $S(p_t)$ is a weak best response.

2. $\underline{s_t \in \{s_0, s'\}}$: As in the analysis of equilibrium $\mathcal{E}_\Delta$, it is sufficient to condition on the state s_0. Since $t < \hat{t}_\Delta$, the equilibrium strategy of firm A is to offer 0. Playing the equilibrium strategy thus results in a payoff to firm A of

$$\mathbb{E}_t \left[e^{-(r+\lambda)(\hat{t}_\Delta - t)} \max\{0, S(p_{\hat{t}_\Delta}) - \hat{w}_\Delta(p_{\hat{t}_\Delta})\} \right].$$

A one-stage deviation to any offer $w_A < \underline{w}_\Delta(p_t, s_0)$ results in exactly the same payoff while a one-stage deviation to an offer $w_A \geqslant \underline{w}_\Delta(p_t, s_0)$ yields in a payoff of

$$S(p_t) - w_A \leqslant S(p_t) - \underline{w}_\Delta(p_t, s_0).$$

To prove that $S(p_t) - \underline{w}_\Delta(p_t, s_0) \leqslant \mathbb{E}_t \left[e^{-(r+\lambda)(\hat{t}_\Delta - t)} \max\{0, S(p_{\hat{t}_\Delta}) - \hat{w}_\Delta(p_{\hat{t}_\Delta})\} \right]$, first observe that for all τ,

$$V_\Delta(p_\tau) > \overline{w}_\Delta(p_\tau, s') \geqslant \mathbb{E}_t \left[e^{-r\Delta} \overline{w}_\Delta(p_{\tau+\Delta}, s') \right].$$

Then applying Lemma 4, there exists some $\bar{t}_\Delta$ such that

$$\begin{cases} W_\Delta(p_\tau, \overline{w}_\Delta(\cdot, s')) - S(p_\tau) > 0 & \text{if } \tau < \bar{t}_\Delta, \\ W_\Delta(p_\tau, \overline{w}_\Delta(\cdot, s')) - S(p_\tau) < 0 & \text{if } \tau \geqslant \bar{t}_\Delta. \end{cases}$$

First note that $\bar{t}_\Delta \leqslant \hat{t}_\Delta$ since for all $\tau \geqslant \bar{t}_\Delta$, $\underline{w}_\Delta(p_\tau, s_0) = \hat{w}_\Delta(p_\tau)$. Furthermore, for $\tau = \hat{t}_\Delta - \Delta$,

$$W_\Delta(p_{\hat{t}_\Delta - \Delta}, \overline{w}_\Delta(\cdot, s_0)) = W_\Delta(p_{\hat{t}_\Delta - \Delta}, \hat{w}_\Delta),$$

which implies that $W_\Delta(p_\tau, \overline{w}_\Delta(\cdot, s_0)) - S(p_\tau) > 0$. Thus, $\bar{t}_\Delta = \hat{t}_\Delta$.

Given this, note that

$$
\begin{aligned}
S(p_t) - \underline{w}_\Delta(p_t, s_0) &< W_\Delta(p_t, \overline{w}_\Delta(\cdot, s')) - \underline{w}_\Delta(p_t, s_0) \\
&= \mathbb{E}_t \left[e^{-(r+\lambda)\Delta} \max\{0, S(p_{t+\Delta}) - \underline{w}_\Delta(p_{t+\Delta}, s_0)\} \right] \\
&< \cdots < \mathbb{E}_t \left[e^{-(r+\lambda)(\hat{t}_\Delta - t)} \max\{0, S(p_{\hat{t}_\Delta}) - \hat{w}_\Delta(p_{\hat{t}_\Delta})\} \right].
\end{aligned}
$$

Finally we show that firm B's strategy is optimal.

1. $\underline{s_t = s''}$: The argument here is the same as for Firm A.

2. $\underline{s_t = s'}$: Suppose that $w_A \geqslant \underline{w}_\Delta(p_t, s')$. Then it is clear that the best response is $\min\{w_A, S(p_t)\}$. Suppose instead that $w_A < \underline{w}_\Delta(p_t, s')$. By playing the equilibrium strategy, firm B obtains a payoff of:

$$
\mathbb{E}_t \left[e^{-r(\hat{t}_\Delta - t)} \max\{0, S(p_{\hat{t}_\Delta}) - \hat{w}_\Delta(p_{\hat{t}_\Delta})\} \right].
$$

Note that a one-stage deviation of offering $w_B < \underline{w}_\Delta(p_t, s')$ yields the same payoff. Any offer $w_B \geqslant \underline{w}_\Delta(p_t, s')$ yields a payoff of:

$$
\begin{aligned}
S(p_t) - w_B \leqslant S(p_t) - \underline{w}_\Delta(p_t, s') &= S(p_t) - \mathbb{E}_t \left[e^{-r(\hat{t}_\Delta - t)} \hat{w}_\Delta(p_{\hat{t}_\Delta}) \right] \\
&< \mathbb{E}_t \left[e^{-r(\hat{t}_\Delta - t)} \max\{0, S(p_{\hat{t}_\Delta}) - \hat{w}_\Delta(p_{\hat{t}_\Delta})\} \right],
\end{aligned}
$$

where the last inequality follows from the fact that $t < \hat{t}_\Delta \leqslant t^*_\Delta$. As a result, an offer of $\min\{w_A, S(p_t)\}$ is optimal for firm B.

■

D Proofs of Section 6

Fix any equilibrium. Let us define the following continuation values. Given any equilibrium, let $\overline{w}_\Delta(p_t, s)$ denote the worker's continuation value at time t conditional on no slip ups and state s. Similarly, let $\underline{w}_\Delta(p_t, s)$ be the worker's continuation value at time t conditional on state s (conditional on no slip-up having occurred before that time) to rejecting all offers at time t and playing his equilibrium strategy from time $t + \Delta$ on. Because the continuation values are associated with an equilibrium, clearly, for all s, $\overline{w}_\Delta(p_t, s) \geqslant \underline{w}_\Delta(p_t, s)$. Since after a breakdown, the belief stays at 0 forever, the continuation values are such that for all s,

$$\underline{w}_\Delta(0, s) = \overline{w}_\Delta(0, s) = 0.$$

We first begin with lemmas that establish continuation values for the players in states s'' and s'.

Lemma 5. *In any equilibrium, for all t, $\overline{w}_\Delta(p_t, s'') = V_\Delta(p_t)$. Consequently, at any time t conditional on state s'', both firms obtain a continuation value of* 0*.*

Proof: Consider any time $t \geqslant t^*_\Delta$. Suppose by contradiction that $\overline{w}_\Delta(p_t, s'') < V_\Delta(p_t)$. Note that $\overline{w}_\Delta(p_t, s'') \geqslant \underline{w}_\Delta(p_t, s'') = e^{-r\Delta}\mathbb{E}_t\left[\overline{w}_\Delta(p_{t+\Delta}, s'')\right]$. First suppose that

$$\max\{w_A(p_t, s''), w_B(p_t, s'')\} > \underline{w}_\Delta(p_t, s''),$$

in which case, $\max\{w_A(p_t, s''), w_B(p_t, s'')\} = \overline{w}_\Delta(p_t, s'') < S(p_t) = V_\Delta(p_t)$. But in this scenario, at least one firm has an incentive to offer $\max\{w_A(p_t, s''), w_B(p_t, s'')\} + \varepsilon$ for $\varepsilon > 0$ sufficiently small, which is a contradiction.

Secondly suppose that

$$\max\{w_A(p_t, s''), w_B(p_t, s'')\} \leqslant \underline{w}_\Delta(p_t, s''),$$

so that $\overline{w}_\Delta(p_t, s'') = \underline{w}_\Delta(p_t, s'')$. Then the worker will indeed accept all offers strictly above $\underline{w}_\Delta(p_t, s'')$. Thus, either firm by offering $\underline{w}_\Delta(p_t, s'') + \varepsilon$ will obtain a payoff of $S(p_t) - \underline{w}_\Delta(p_t, s'') - \varepsilon$. On the other hand, by waiting until at least $t + \Delta$, the maximum payoff that either firm will obtain would be

$$e^{-r\Delta}\mathbb{E}_t\left[V_\Delta(p_\tau) - \overline{w}_\Delta(p_\tau, s'')\right]$$

since the firm receives at most the residual of the surplus that is not captured by the worker. But

$$e^{-r\Delta}\mathbb{E}_t\left[V_\Delta(p_{t+\Delta}) - \overline{w}_\Delta(p_{t+\Delta}, s'')\right] = e^{-r\Delta}\mathbb{E}_t\left[S(p_{t+\Delta}\right] - \overline{w}_\Delta(p_t, s'') < S(p_t) - \overline{w}_\Delta(p_t, s'') - \varepsilon$$

for $\varepsilon > 0$ sufficiently small. This contradicts the optimality of the firms' strategies. Thus, we have shown that at all times $t \geqslant t^*_\Delta$, $\overline{w}_\Delta(p_t, s'') = V_\Delta(p_t)$. Clearly, this leaves both firms with zero surplus.

Given the above, the worker at any time $t < t^*_\Delta$ can guarantee the payoff of

$$V_\Delta(p_t) = e^{-rt^*_\Delta}\mathbb{E}_t\left[\max\{0, S(p_{t^*_\Delta})\}\right]$$

by rejecting all offers until t^*_Δ. Since this is the maximum possible payoff that the worker can obtain, this is indeed his equilibrium continuation value. Again, this leaves both firms with continuation payoffs of 0. ■

Lemma 6. *In any equilibrium, if at time t, the state is $s_t = s'$, then firm A's continuation value is* 0.

Proof: Consider any time t at state $s_t = s'$. Suppose that the wage offered at time t and state $s_t = s'$ is w_A.

1. $\underline{w_A \geqslant S(p_t)}$: In this case, since $S(p_t) > \hat{w}_\Delta(p_t) \geqslant \overline{w}_\Delta(p_t, s') \geqslant \underline{w}_\Delta(p_t, s')$, the worker will accept one of the offers. But then firm A receives a payoff of 0 in this case.

2. $\underline{w_A < S(p_t)}$: Suppose by way of contradiction that the worker accepts firm A's offer. Let w_B be the counteroffer of firm B to w_A in equilibrium. Then $w_A \geqslant \max\{w_B, \underline{w}_\Delta(p_t, s')\}$. In this case, firm B receives a payoff of 0 by offering w_B. However, by offering $w_A + \varepsilon$, he receives a payoff of $S(p_t) - w_A - \varepsilon$. Clearly the latter is strictly positive when $\varepsilon > 0$ is sufficiently small, yielding a contradiction. Thus we have shown that all times in state s', either the worker accepts firm B's offer or no hiring occurs.

As a result, the continuation payoff of firm A after state s' must be exactly zero at all times in any equilibrium. ■

We also prove the following lemmas that establish upper bounds on continuation values of the worker in states s_0, s' in any equilibrium.

Lemma 7. *For all t, $\overline{w}_\Delta(p_t, s') \leqslant \overline{w}_\Delta(p_t, s_0)$.*

Proof: Note that by Lemma 5, the continuation value to the worker at time t at state $s_t = s''$ is $V_\Delta(p_t)$. Thus, at time t in state $s_t = s_0$, by waiting a period, the worker can guarantee himself a payoff of

$$e^{-r\Delta}\mathbb{E}_t\Big[e^{-\lambda\Delta}\overline{w}_\Delta(p_{t+\Delta}, s_0) + \left(1 - e^{-\lambda\Delta}\right)\left(\beta\overline{w}_\Delta(p_{t+\Delta}, s') + (1-\beta)V_\Delta(p_{t+\Delta})\right)\Big].$$

Thus,

$$\overline{w}_\Delta(p_t, s_0) \geqslant e^{-r\Delta}\mathbb{E}_t\Big[e^{-\lambda\Delta}\overline{w}_\Delta(p_{t+\Delta}, s_0) + \left(1 - e^{-\lambda\Delta}\right)\left(\beta\overline{w}_\Delta(p_{t+\Delta}, s') + (1-\beta)V_\Delta(p_{t+\Delta})\right)\Big]. \tag{10}$$

When the state is s', firm B will never offer a wage strictly greater than

$$\max\{\overline{w}_\Delta(p_t, s_0), e^{-r\Delta}\mathbb{E}_t\left[\overline{w}_\Delta(p_{t+\Delta}, s')\right]\}$$

since all such offers will be accepted. Therefore, we have the following bound:

$$\overline{w}_\Delta(p_t, s') \leqslant \max\{\overline{w}_\Delta(p_t, s_0), e^{-r\Delta}\mathbb{E}_t\left[\overline{w}_\Delta(p_{t+\Delta}, s')\right]\}.$$

Suppose by way of contradiction that $\overline{w}_\Delta(p_t, s') > \overline{w}_\Delta(p_t, s_0)$. In this case, using (10) and the fact that $\overline{w}_\Delta(p_{t+\Delta}, s') \leqslant V_\Delta(p_\tau)$, we must have

$$\begin{aligned} 0 < \overline{w}_\Delta(p_t, s') - \overline{w}_\Delta(p_t, s_0) &\leqslant e^{-r\Delta}\mathbb{E}_t\left[\overline{w}_\Delta(p_{t+\Delta}, s')\right] - \overline{w}_\Delta(p_t, s_0) \\ &\leqslant e^{-(r+\lambda)\Delta}\left(p_t + (1-p_t)e^{-\eta\Delta}\right)\left[\overline{w}_\Delta(p_{t+\Delta}, s') - \overline{w}_\Delta(p_{t+\Delta}, s_0)\right]. \end{aligned}$$

Iterating this argument, we see that for all $k = 1, 2, \ldots$,

$$0 < \overline{w}_\Delta(p_t, s') - \overline{w}_\Delta(p_t, s_0) < e^{-(r+\lambda)k\Delta}\left(p_t + (1-p_t)e^{-\eta k\Delta}\right)\left[\overline{w}_\Delta(p_{t+k\Delta}, s') - \overline{w}_\Delta(p_{t+k\Delta}, s_0)\right].$$

But the latter converges to zero as $k \to 0$ which is a contradiction. ■

Lemma 8. *In any equilibrium,* $\overline{w}_\Delta(p_t, s'), \overline{w}_\Delta(p_t, s_0) \leqslant \hat{w}_\Delta(p_t)$.

Proof: Consider an arbitrary equilibrium. Because $\overline{w}_\Delta(p_t, s') \leqslant \overline{w}_\Delta(p_t, s_0)$ by Lemma 7, it suffices to prove that $\overline{w}_\Delta(p_t, s_0) \leqslant \hat{w}_\Delta(p_t)$.

Using the same argument as in the first part of the proof of Lemma 7,

$$\overline{w}_\Delta(p_t, s_0) \geqslant e^{-r\Delta}\mathbb{E}_t\Big[e^{-\lambda\Delta}\overline{w}_\Delta(p_{t+\Delta}, s_0) + \left(1 - e^{-\lambda\Delta}\right)\left(\beta\overline{w}_\Delta(p_{t+\Delta}, s') + (1-\beta)V_\Delta(p_{t+\Delta})\right)\Big]$$

Moreover, consider firm A's optimal strategy. Since after states s', s'', its continuation value is zero, in choosing its best response, it is without loss of generality to condition on the event

s_0. Note that conditional on the state s_0, the worker accepts all offers w_A strictly above

$$e^{-r\Delta}\mathbb{E}_t\Big[e^{-\lambda\Delta}\overline{w}_\Delta(p_{t+\Delta},s_0)+\left(1-e^{-\lambda\Delta}\right)\left(\beta\overline{w}_\Delta(p_{t+\Delta},s')+(1-\beta)V_\Delta(p_{t+\Delta})\right)\Big].$$

As a result, firm A will never offer a wage w_A strictly above this quantity. Together with the inequality above, this implies that

$$\overline{w}_\Delta(p_t,s_0)=e^{-r\Delta}\mathbb{E}_t\left[e^{-\lambda\Delta}\overline{w}_\Delta(p_{t+\Delta},s_0)+\left(1-e^{-\lambda\Delta}\right)\left(\beta\overline{w}_\Delta(p_{t+\Delta},s')+(1-\beta)V_\Delta(p_{t+\Delta})\right)\right].$$

Using the fact that $\overline{w}_\Delta(p_{t+\Delta},s')\leqslant\overline{w}_\Delta(p_{t+\Delta},s_0)$, we obtain:

$$\overline{w}_\Delta(p_t,s_0)\leqslant e^{-r\Delta}\mathbb{E}_t\Big[\left(e^{-\lambda\Delta}+\left(1-e^{-\lambda\Delta}\right)\beta\right)\overline{w}_\Delta(p_{t+\Delta},s_0)+(1-\beta)\left(1-e^{-\lambda\Delta}\right)V_\Delta(p_{t+\Delta})\Big].$$

Thus, using the recursive definition of $\hat{w}_\Delta$ given in (7),

$$\overline{w}_\Delta(p_t,s_0)-\hat{w}_\Delta(p_t)\leqslant e^{-r\Delta}\left(e^{-\lambda\Delta}+\left(1-e^{-\lambda\Delta}\right)\beta\right)\mathbb{E}_t\left[\overline{w}_\Delta(p_{t+\Delta},s_0)-\hat{w}_\Delta(p_{t+\Delta})\right]$$

Iterating, we obtain for all $k=1,2,\ldots$,

$$\overline{w}_\Delta(p_t,s_0)-\hat{w}_\Delta(p_t)\leqslant e^{-rk\Delta}\left(e^{-\lambda\Delta}+\left(1-e^{-\lambda\Delta}\right)\beta\right)^k\mathbb{E}_t\left[\overline{w}_\Delta(p_{t+k\Delta},s_0)-\hat{w}_\Delta(p_{t+k\Delta})\right].$$

The right hand side converges to zero as $k\to\infty$, which implies that $\overline{w}_\Delta(p_t,s_0)\leqslant\hat{w}_(p_t)$. ■

D.1 Proof of Theorem 1

D.1.1 Equilibrium Behavior at $t\geqslant\hat{t}_\Delta$

We first analyze the equilibrium behavior at $t\geqslant\hat{t}_\Delta$.

Lemma 9. *At all times $t\geqslant\hat{t}_\Delta$, conditional on no slip-ups and $s_t=s_0$, the worker accepts the equilibrium offered wage of firm A with probability one.*

Proof: We must have:

$$\begin{aligned}
&\overline{w}_\Delta(p_t,s_0)\\
&\leqslant\mathbb{E}_t\left[e^{-r\Delta}\left(e^{-\lambda\Delta}\overline{w}_\Delta(p_{t+\Delta},s_0)+\left(1-e^{-\lambda\Delta}\right)\beta\overline{w}_\Delta(p_{t+\Delta},s')+\left(1-e^{-\lambda\Delta}\right)(1-\beta)V_\Delta(p_{t+\Delta})\right)\right]\\
&\leqslant\mathbb{E}_t\left[e^{-r\Delta}\left(\left(e^{-\lambda\Delta}+\left(1-e^{-\lambda\Delta}\right)\beta\right)\overline{w}_\Delta(p_{t+\Delta},s_0)+\left(1-e^{-\lambda\Delta}\right)(1-\beta)V_\Delta(p_{t+\Delta})\right)\right].
\end{aligned}$$

Otherwise, an agreement would have to occur at time t at the wage $\overline{w}_\Delta(p_t, s_0)$. However in such a scenario, note that the worker will accept any offer strictly above

$$e^{-r\Delta}\mathbb{E}_t\left[\left(e^{-\lambda\Delta} + \left(1 - e^{-\lambda\Delta}\right)\beta\right)\overline{w}_\Delta(p_{t+\Delta}, s_0) + \left(1 - e^{-\lambda\Delta}\right)(1-\beta)V_\Delta(p_{t+\Delta})\right],$$

which means that there exists some $\varepsilon > 0$ such that the worker will still accept $\overline{w}_\Delta(p_t, s_0) - \varepsilon$ with probability one. But this contradicts the fact that firm A is best responding.

Given the above inequality, and using (7), we then have:

$$\begin{aligned}\hat{w}_\Delta(p_t) - \overline{w}_\Delta(p_t, s_0) &\geqslant e^{-r\Delta}\left(e^{-\lambda\Delta} + \left(1 - e^{-\lambda\Delta}\right)\beta\right)\mathbb{E}_t\left[\hat{w}_\Delta(p_{t+\Delta}) - \overline{w}_\Delta(p_{t+\Delta}, s_0)\right] \\ &\geqslant e^{-(r+\lambda)\Delta}\mathbb{E}_t\left[\hat{w}_\Delta(p_{t+\Delta}) - \overline{w}_\Delta(p_{t+\Delta}, s_0)\right],\end{aligned}$$

where the second inequality follows from Lemma 8. Furthermore, by the definition of $\hat{t}_\Delta$,

$$S(p_t) - \hat{w}_\Delta(p_t) > e^{-(r+\lambda)\Delta}\mathbb{E}_t\left[\max\{0, S(p_{t+\Delta})\} - \hat{w}_\Delta(p_{t+\Delta})\right].$$

Adding these inequalities, we have:

$$S(p_t) - \overline{w}_\Delta(p_t, s_0) > e^{-(r+\lambda)\Delta}\mathbb{E}_t\left[\max\{0, S(p_{t+\Delta})\} - \overline{w}_\Delta(p_{t+\Delta}, s_0)\right].$$

Iterating, we obtain for all $k = 1, 2, \ldots$,

$$S(p_t) - \overline{w}_\Delta(p_t, s_0) > e^{-(r+\lambda)k\Delta}\mathbb{E}_t\left[\max\{0, S_{t+k\Delta}\} - \overline{w}_\Delta(p_{t+k\Delta}, s_0)\right].$$

But this implies that an agreement must occur at time t. ■

Given the above, equilibria take a very simple structure at all times $t \geqslant \hat{t}_\Delta$. Furthermore note that if acceptance occurs at all such times, then the continuation value at time t conditional on an unobserved arrival by firm B is exactly equal to the continuation value at time t conditional on no arrivals by firm B. This observation allows us to obtain the following proposition.

Lemma 10. *At all times $t \geqslant \hat{t}_\Delta$, conditional on no slip ups and no arrival by firm B, firm A hires the worker at the wage $\hat{w}_\Delta(p_t)$ with probability one.*

Proof: We will show that indeed $\overline{w}_\Delta(p_t, s_0) = \hat{w}_\Delta(p_t)$ for all $t \geqslant \hat{t}_\Delta$. To see this note that using the same argument for the first part of proof of the previous lemma,

$$\overline{w}_\Delta(p_t, s_0) \leqslant e^{-r\Delta}\mathbb{E}_t\left[\left(e^{-\lambda\Delta} + \left(1 - e^{-\lambda\Delta}\right)\beta\right)\overline{w}_\Delta(p_{t+\Delta}, s_0) + \left(1 - e^{-\lambda\Delta}\right)(1-\beta)V_\Delta(p_{t+\Delta})\right].$$

Furthermore, since we know by the previous lemma that agreement must be reached at period $t + \Delta$, by waiting until the next period, the worker can always guarantee the payoff of

$$e^{-r\Delta}\mathbb{E}_t\left[\left(e^{-\lambda\Delta} + \left(1 - e^{-\lambda\Delta}\right)\beta\right)\overline{w}_\Delta(p_{t+\Delta}, s_0) + \left(1 - e^{-\lambda\Delta}\right)(1-\beta)V_\Delta(p_{t+\Delta})\right].$$

Therefore, for all t,

$$\overline{w}_\Delta(p_t, s_0) = e^{-r\Delta}\mathbb{E}_t\left[\left(e^{-\lambda\Delta} + \left(1 - e^{-\lambda\Delta}\right)\beta\right)\overline{w}_\Delta(p_{t+\Delta}, s_0) + \left(1 - e^{-\lambda\Delta}\right)(1-\beta)V_\Delta(p_{t+\Delta})\right].$$

But note that $\hat{w}_\Delta$ satisfies the same difference equation.

Therefore we have:

$$\overline{w}_\Delta(p_t, s_0) - \hat{w}_\Delta(p_t) = e^{-r\Delta}\left(e^{-\lambda\Delta} + \left(1 - e^{-\lambda\Delta}\right)\beta\right)\mathbb{E}_t\left[\overline{w}_\Delta(p_{t+\Delta}, s_0) - \hat{w}_\Delta(p_{t+\Delta})\right]$$

Iterating, we have for all $k = 1, 2, \ldots$,

$$\overline{w}_\Delta(p_t, s_0) - \hat{w}_\Delta(p_t) = e^{-rk\Delta}\left(e^{-\lambda\Delta} + \left(1 - e^{-\lambda\Delta}\right)\beta\right)^k\mathbb{E}_t\left[\overline{w}_\Delta(p_{t+k\Delta}, s_0) - \hat{w}_\Delta(p_{t+k\Delta})\right].$$

This implies that $\overline{w}_\Delta(p_t, s_0) - \hat{w}_\Delta(p_t) = 0$ since otherwise,

$$\lim_{k\to\infty}\left|\overline{w}_\Delta(p_{t+k\Delta}, s_0) - \hat{w}_\Delta(p_{t+k\Delta})\right| = +\infty.$$

This concludes the proof. ■

D.1.2 Equilibrium Behavior at $t < \hat{t}_\Delta$

We will show that at all such times, firm A does not hire the worker.

Lemma 11. *Suppose that $t < \hat{t}_\Delta$. Then in any equilibrium, conditional on no slip-ups and no arrivals by firm B, the offered wage of firm A is rejected with probability one.*

Proof: To see this, define the value function $z(\cdot, s')$ as follows. First define $z(0, s') = 0$ and define:

$$z_\Delta(p_t, s') = \begin{cases} e^{-r(\hat{t}_\Delta - t)}\mathbb{E}_t\left[\hat{w}_\Delta(p_{\hat{t}_\Delta})\right] & \text{if } t < \hat{t}_\Delta \\ \hat{w}_\Delta(p_t) & \text{if } t \geqslant \hat{t}_\Delta. \end{cases}$$

Furthermore, define $z_\Delta(\cdot, s_0)$ as follows. Define $z_\Delta(0, s_0) = 0$ and recursively define:

$$z_\Delta(p_t, s_0) = e^{-r\Delta}\mathbb{E}_t\left[e^{-\lambda\Delta} z_\Delta(p_{t+\Delta}, s_0) + \left(1 - e^{-\lambda\Delta}\right)\left(\beta z_\Delta(p_{t+\Delta}, s') + (1-\beta)V_\Delta(p_{t+\Delta})\right)\right].$$

Note that $z_\Delta(p_t, s')$ is a lower bound on the continuation payoff that a worker can guarantee at time t at state s' in *any equilibria*. This is because the worker knows that $\hat{w}_\Delta(p_\tau)$ will be offered for sure at time $\tau = \hat{t}_\Delta$ by Lemma 10. Thus the strategy of rejecting all offers until time $\hat{t}_\Delta$ delivers the payoff of $z_\Delta(p_t, s')$. This then implies that $z_\Delta(p_t, s_0)$ is a lower bound on the continuation payoff that a worker can guarantee for himself conditional on no arrivals. As a result, all offers strictly below $z_\Delta(p_t, s_0)$ are rejected in all equilibria. Thus at time t, conditional on state s_0, the most that firm A can obtain if an agreement is reached is $S(p_t) - z_\Delta(p_t, s_0)$.

Now let us examine the incentives of firm A. Since we showed previously that in any equilibrium, firm A's continuation value after an arrival of firm B is 0, it is without loss of generality to condition on the event s_0. Clearly, $S(p_{\hat{t}_\Delta - \Delta}) < W_\Delta(p_{\hat{t}_\Delta - \Delta}, z_\Delta(\cdot, s'))$ since $z_\Delta(p_\tau, s') = \hat{w}_\Delta(p_\tau)$ for all $\tau \geqslant \hat{t}_\Delta$. This together with Lemma 4 implies that for all $t < \hat{t}_\Delta$, $S(p_t) < W_\Delta(p_t, z_\Delta(\cdot, s'))$. Therefore, for all $t < \hat{t}_\Delta$,

$$S_t - z_\Delta(p_t, s_0) < e^{-(r+\lambda)\Delta}\mathbb{E}_t\left[\max\{0, S(p_{t+\Delta}) - z_\Delta(p_{t+\Delta}, s_0)\}\right].$$

Iterating, we obtain:

$$\begin{aligned} S_t - z_\Delta(p_t, s_0) &< e^{-(r+\lambda)(\hat{t}_\Delta - t)}\mathbb{E}_t\left[\max\{0, S(p_{\hat{t}_\Delta}) - z_\Delta(p_{\hat{t}_\Delta}, s_0)\}\right] \\ &= e^{-(r+\lambda)(\hat{t}_\Delta - t)}\mathbb{E}_t\left[\max\{0, S(p_{\hat{t}_\Delta}) - \hat{w}_\Delta(p_{\hat{t}_\Delta})\}\right]. \end{aligned}$$

As a result, it is never incentive compatible for firm A to hire the worker at time t. ■

All together these lemmas imply Theorem 1.

D.2 Proof of Theorem 2

Proof of Theorem 2: First note that firm A only receives positive surplus when there are no arrivals of firm B before $\hat{t}_\Delta$. In the event that firm B does not arrive before $\hat{t}_\Delta$, in any equilibrium, firm A hires the worker for a wage of $\hat{w}_\Delta(\hat{p}_\Delta)$ at time $\hat{t}_\Delta$ (conditional on no slip-ups). As a result, firm A's payoff is

$$e^{-(r+\lambda)\hat{t}_\Delta}\mathbb{E}_0\left[\max\{0, S(p_{\hat{t}_\Delta}) - \hat{w}_\Delta(p_{\hat{t}_\Delta})\}\right]$$

in any equilibrium. This shows that all equilibria are payoff-equivalent for firm A.

To prove point 4, note that efficiency is maximized when the time of hiring is as close to t^*_Δ as possible. By Theorem 1, we know that hiring will take place at least before $\hat{t}_\Delta$ in states s_0 and s' in all equilibria. Thus clearly $\mathcal{E}^*_\Delta$ is the most efficient equilibrium. By the same token, note that $\mathcal{E}_\Delta$ is the least efficient equilibrium.

To analyze the payoffs of firm B and the worker, let Π_A, Π_B, and Π_w denote the corresponding payoffs of the players in a particular equilibrium.

By the previous observations, Π_A is constant across all equilibria. Thus if there is an equilibrium that simultaneously minimizes efficiency and maximizes the worker's payoff, this equilibrium will also minimize Π_B. We will show that indeed $\mathcal{E}_\Delta$ achieves this. To see this, note that by Lemma 8, the worker's payoffs are bounded above by $\hat{w}_\Delta(p_0)$. Thus, Π_w is maximized in equilibrium $\mathcal{E}_\Delta$. Furthermore, we previously observed that efficiency is minimized in equilibrium $\mathcal{E}_\Delta$. This then implies that Π_B is minimized in equilibrium $\mathcal{E}_\Delta$.

Similarly, by Theorem 1, we know that in all equilibria, in state s', firm A offers $\hat{w}_\Delta(p_t)$ for all $p_t \geqslant \hat{p}$. It ensues that Π_w is minimized in equilibrium $\mathcal{E}^*_\Delta$. Furthermore this is the equilibrium that simultaneously maximizes efficiency and so Π_B must be maximized in $\mathcal{E}^*_\Delta$. ■

E Proofs of Section 7

E.1 Limit Results: $\Delta \to 0$

First, we show the convergence of first-best stopping times and beliefs.

Lemma 12. *For all $p \in (0,1)$, $t^*(p) = \lim_{\Delta\to 0} t^*_\Delta(p)$ and thus $p^* = \lim_{\Delta\to 0} p^*_\Delta(p)$.*

Proof: Note that t^*_Δ satisfies the following set of inequalities:

$$e^{-r\Delta}\mathbb{E}_{t^*_\Delta-\Delta}\left[\max\{0,S(p_{t^*_\Delta})\}-S(p_{t^*_\Delta-\Delta})\right]>0>e^{-r\Delta}\mathbb{E}_{t^*_\Delta}\left[\max\{0,S(p_{t^*_\Delta+\Delta})\}-S(p_{t^*_\Delta})\right].$$

Rewriting the above and dividing by $\Delta>0$, we obtain:

$$\begin{aligned}&e^{-r\Delta}c(1-p_{t^*_\Delta-\Delta})\frac{1-e^{-\eta\Delta}}{\Delta}-\frac{1-e^{-r\Delta}}{\Delta}S(p_{t^*_\Delta-\Delta})\\&>0>e^{-r\Delta}c(1-p_{t^*_\Delta})\frac{1-e^{-\eta\Delta}}{\Delta}-\frac{1-e^{-r\Delta}}{\Delta}S(p_{t^*_\Delta}).\end{aligned}$$

But the above implies that $\lim_{\Delta\to 0}c(1-p_{t^*_\Delta})\eta-rS(p_{t^*_\Delta})=0$. This implies that $\lim_{\Delta\to 0}t^*_\Delta=t^*$. ■

In order to show convergence of $\hat{p}_\Delta$, we need to establish uniform convergence of V_Δ and $\hat{w}_\Delta$.

Lemma 13. *As $\Delta\to 0$, $V_\Delta(p)\to V(p)$ and $\hat{w}_\Delta(p)\to\hat{w}(p)$ uniformly for all p.*

Proof: Note that the function $T\mapsto\mathbb{E}e^{-rT}\left[\max\{0,S(p_T)\}|p_0=p\right]$ is continuous, hence, uniformly continuous on bounded intervals. Moreover, the limit $\lim_{T\to\infty}\mathbb{E}e^{-rT}\left[\max\{0,S(p_T)\}|p_0=p\right]=0$. Therefore, $\lim_{\Delta\to 0}V_\Delta(p)=V(p)$ point-wise. By Dini's theorem, if we consider the sequence $\Delta,\frac{\Delta}{2},\ldots$, i.e., we cut the time intervals in half at every step, then the convergence must be uniform because p is in a compact set, V is continuous in p, and $V_\Delta(p)$ is continuous in p and decreasing in Δ.

Recall that by (2) $\hat{w}_\Delta(p)=\mathbb{E}[e^{-r\hat{T}}V_\Delta(p_{\hat{T}})|p_0=p]$ where $\hat{T}/\Delta$ is geometrically distributed with success rate $(1-\beta)(1-e^{-\lambda\Delta})$. We can write

$$\mathbb{E}[e^{-r\hat{T}}V_\Delta(p_{\hat{T}})|p_0=p]=\mathbb{E}\left[\sum_{i=0}^{\infty}\Delta\frac{Pr(\hat{T}=i\Delta)}{\Delta}e^{-ri\Delta}V_\Delta(p_{i\Delta})\middle|p_0=p\right]$$

Then, almost surely, $\frac{Pr(\hat{T}=t)}{\Delta}\cdot e^{-rt}V_\Delta(p_t)$ converges uniformly in t to $e^{-(1-\beta)\lambda t}(1-\beta)\lambda\cdot e^{-rt}V(p_t)$ because the path $t\mapsto p_t$ converges uniformly in t almost surely and V_Δ converges uniformly in p. Thus, $\sum_{i=0}^{\infty}\Delta\frac{Pr(\hat{T}=i\Delta)}{\Delta}e^{-ri\Delta}V_\Delta(p_{i\Delta})$ converges almost surely to $\int_0^\infty e^{-(1-\beta)\lambda t}(1-\beta)\lambda e^{-rt}V(p_t)\,dt$. Then, the convergence of $\hat{w}_\Delta$ follows from the bounded convergence theorem since $V_\Delta(p),V(p)<b$. ■

Finnaly, we show convergence of $\hat{t}$ and $\hat{p}$.

Lemma 14. $\lim_{\Delta\to 0}\hat{t}_\Delta = \hat{t}$ *and hence* $\lim_{\Delta\to 0}\hat{p}_\Delta = \hat{p}$.

Proof: Recall that $\hat{t}_\Delta \in \mathbb{T}_\Delta$ is the unique time such that

$$\begin{cases} \mathbb{E}_t\left[e^{-(r+\lambda)\Delta}\max\{0, S(p_{t+\Delta}) - \hat{w}_\Delta(p_{t+\Delta})\}\right] > S(p_t) - \hat{w}_\Delta(p_t) & \text{if } t < \hat{t}_\Delta \\ \mathbb{E}_t\left[e^{-(r+\lambda)\Delta}\max\{0, S(p_{t+\Delta}) - \hat{w}_\Delta(p_{t+\Delta})\}\right] < S(p_t) - \hat{w}_\Delta(p_t) & \text{if } t \geqslant \hat{t}_\Delta. \end{cases}$$

Rewriting this condition gives us that $\hat{t}_\Delta$ is the smallest t such that

$$\mathbb{E}_t\left[e^{-(r+\lambda)\Delta}\left(\frac{S(p_{t+\Delta}) - S(p_t)}{\Delta} + \frac{\max\{0, S(p_{t+\Delta})\} - S(p_{t+\Delta})}{\Delta} - \frac{\hat{w}_\Delta(p_{t+\Delta}) - \hat{w}_\Delta(p_t)}{\Delta}\right)\right] \\ + \frac{1 - e^{-(r+\lambda)\Delta}}{\Delta}(\hat{w}(p_t) - S(p_t)) \qquad < 0$$

Note that $\lim_{\Delta\to 0}\frac{S(p_{t+\Delta})-S(p_t)}{\Delta} = (b+c)\dot{p}_t$ and

$$\begin{aligned} \lim_{\Delta\to 0}\frac{\hat{w}_\Delta(p_{t+\Delta}) - \hat{w}_\Delta(p_t)}{\Delta} &= \lim_{\Delta\to 0}\hat{w}_\Delta(p_{t+\Delta})\frac{1 - e^{-(r+\lambda)\Delta}(p_t + e^{-\eta\Delta}(1-p_t))}{\Delta} + \\ &\quad \frac{1 - e^{-\lambda\Delta}}{\Delta}e^{-r\Delta}(\beta\hat{w}_\Delta(p_{t+\Delta}) + (1-\beta)V_\Delta(p_{t+\Delta})) \\ &= \hat{w}(p_t)(r + \lambda + \eta(1-p_t)) + \lambda(\beta\hat{w}(p_t) + (1-\beta)V(p_t)). \end{aligned}$$

by Lemma 13. Moreover, $\max\{0, S(p_{t+\Delta})\} - S(p_{t+\Delta}) = 0$ as long as $S(p_{t+\Delta}) \geqslant 0$. Thus, the definition of $p_{\hat{t}_\Delta}$ implies that as $\Delta \to 0$, it must solve

$$rS(\hat{p}) = c\eta(1-\hat{p}) - \lambda\Big[S(\hat{p}) - \Big((1-\beta)V(\hat{p}) + \beta\hat{w}(p)\Big)\Big]. \tag{11}$$

This proofs the lemma. ■

E.2 Comparative Statics

Proof of Lemma 3: $\hat{p}$ satisfies:

$$rS(\hat{p}) = c\eta(1-\hat{p}) - \lambda\Big[S(\hat{p}) - \Big((1-\beta)V(\hat{p}) + \beta\hat{w}(p)\Big)\Big]; \tag{12}$$

while p^* satisfies:

$$rS(p^*) = c\eta(1-p^*). \tag{13}$$

For $p \geqslant p^*$, $V(p) = S(p)$. Moreover $\hat{w}(p) < V(p)$, for all p. Thus, at p^* the term in the squared bracket in (12) is positive. This implies that $\hat{p} < p^*$.

That $\hat{p}$ is decreasing in β follows from noting that the term in the squared bracket in (12) is decreasing in β since (a) $\hat{w}$ is decreasing in β by virtue of (6), (b) $\hat{w}(p) < V(p)$ for all p. We are only left to show that $\hat{p}$ tends to p^* if λ tends to either zero or infinity. Setting $\lambda = 0$ in (12) yields (13). This yields one part. For the second part, divide through by λ in (12). In the limit as λ tends to infinity, we obtain $0 = S(\hat{p}) - V(\hat{p})$ since, by (6), $\hat{w}(p)$ tends to $V(p)$ as λ tends to infinity. But $V(p^*) = S(p^*)$. This concludes the second part.

■

Proof of Theorem 3: Immediate from Lemma 3 and the observation that in $\mathcal{E}^*_\Delta$, conditional on no slip-up before then, with probability 1 the worker is hired the instant the belief hits $\hat{p}$.
■

Proof of Proposition 3: We first show that Π^*_A is decreasing in λ. Let $\lambda_2 > \lambda_1$. Note that it is enough to show that for the reservation wages we have $\underline{w}_2(p, s_0) \geqslant \underline{w}_1(p, s_0)$ for all $p \in [\hat{p}_2, \hat{p}_1]$ if $\hat{p}_2 < \hat{p}_1$ or all $p \in [\hat{p}_1, \hat{p}_2]$ if $\hat{p}_1 < \hat{p}_2$. We consider each case in turn.

Case (i): $\hat{p}_2 < \hat{p}_1$. We then have, for all $p \in [\hat{p}_2, \hat{p}_1]$:

$$\underline{w}_1(p, s_0) \leqslant \hat{w}_2(p) = \underline{w}_2(p, s_0).$$

Case (ii): $\hat{p}_1 < \hat{p}_2$. Suppose that we can find $p \in [\hat{p}_1, \hat{p}_2]$ such that $\underline{w}_2(p, s_0) < \underline{w}_1(p, s_0)$. Since $p > \hat{p}_1$, under λ_1, when the belief is p firm A prefers to hire the worker than wait until the belief hits $\hat{p}_2$. The same must therefore be true under λ_2 as well since A's payoff from hiring the worker at p is now higher, whereas A's payoff from waiting until $\hat{p}_2$ is lower. So, under λ_2, when the belief is p firm A prefers to hire the worker than wait until the belief hits $\hat{p}_2$. This contradicts the definition of $\hat{p}_2$.

Next we show that Π^*_w is increasing in λ. As $\hat{t}$ is firm A's optimal hiring time, we have

$$\hat{t} = \arg\max_{t \geqslant 0} e^{-(r+\lambda)t}(p_0 + (1-p_0)e^{-\eta t})(S_t - \hat{w}_t).$$

Let us write the objective function here as $F(t)$.

Consider next the coalition made up of the worker and firm A, under the constraint that passed T_B the worker is hired by firm B. By waiting an instant Δ the coalition guarantees itself

$$S_t + \Big[\dot{S}_t - (r + (1-p_t)\eta)S_t + \lambda\big[\big(\beta \underline{w}_t(s') + (1-\beta)V_t\big) - S_t\big]\Big]\Delta + o(\Delta).$$

This shows that waiting until $\hat{t}$ is the coalition's optimal policy. Hence:

$$\begin{aligned}\hat{t} = \arg\max_{t\geqslant 0} &\int_0^t \lambda e^{-(\lambda+r)\tau}\left(p_0 + (1-p_0)e^{-\eta\tau}\right)\left(\beta \underline{w}_\tau(s') + (1-\beta)V_\tau\right) d\tau \\ &+ e^{-(r+\lambda)t}\left(p_0 + (1-p_0)e^{-\eta t}\right) S_t.\end{aligned}$$

Let us write the objective function above as $H(t)$. Furthermore, observe that

$$\hat{t} = \arg\max_{t\geqslant 0} e^{-rt}\left(p_0 + (1-p_0)e^{-\eta t}\right)\left((1-e^{-\lambda t})(\beta\hat{w}_t + (1-\beta)V_t) + e^{-\lambda t}S_t\right).$$

Let us write the objective function here as $G(t)$. Notice that $G(\hat{t}) = H(\hat{t})$. This allows us to write

$$\Pi^*_w = G(\hat{t}) - F(\hat{t}).$$

By the envelope theorem,

$$\begin{aligned}\frac{d}{d\lambda}\left(G(\hat{t}) - F(\hat{t})\right) &= \frac{\partial}{\partial\lambda}\left(G(\hat{t}) - F(\hat{t})\right) + (G'(\hat{t}) - F'(\hat{t}))\frac{d\hat{t}}{d\lambda} \\ &= \frac{\partial}{\partial\lambda}\left(G(\hat{t}) - F(\hat{t})\right).\end{aligned}$$

But note that for every t,

$$G(t) - F(t) = \left(1 - e^{-\lambda t}\right)\left(\beta V_t + (1-\beta)e^{-rt}(p_0 + (1-p_0)e^{-\eta t})\hat{w}_t\right) + e^{-(r+\lambda)t}(p_0 + (1-p_0)e^{-\eta t})\hat{w}_t,$$

which is increasing in λ. Thus, Π^*_w is increasing in λ.

The proof that Π^*_w is decreasing in β and Π^*_A increasing β rests on arguments similar to those used to show that Π^*_A is decreasing in λ, and is therefore omitted. Finally that Π^*_w tends to W^* as λ tends to infinity, and Π^*_A tends to W^* as λ tends to zero rests on the arguments used to show Lemma 3. When λ tends to zero $\hat{w}(\cdot)$ tends to zero as well. When λ tends to infinity $\hat{w}(\cdot)$ tends to $V(\cdot;(S,r))$, and $\hat{p}$ tends to $p^*(S,r)$. Thus, as λ approaches infinity, firm A offers a wage approaching $S(p^*)$ at a belief approaching p^*.

■